

# Relaxing competition through speculation: Committing to a negative supply slope\*

## DRAFT VERSION

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### Abstract

We analyze how suppliers can take strategic speculative positions in derivatives markets to soften competition in the spot market. In our game, suppliers first choose a portfolio of call options and then compete with supply functions. In equilibrium firms sell forward contracts and buy call options to commit to downward sloping supply functions. This strategy is risky but it makes competitors' residual demand less price sensitive, so they increase their mark-ups. We show that this type of strategic speculation increases the level and volatility of commodity prices and decreases welfare.

**Keywords:** Supply function equilibrium, Option contracts, Strategic commitment, Speculation

**JEL codes:** C73, D43, D44, G13, L13, L94

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# 1 Introduction

The trade in commodity derivatives is widespread and trading volumes often surpass that of the underlying commodities.<sup>1</sup> Ideally derivatives markets improve market efficiency as they allow firms to manage risk and facilitate price discovery by aggregating information across market participants.<sup>2</sup> We show that derivatives trade increases spot market volatility and harms competition, when dominant producers trade futures and option contracts and make price-contingent supply offers in the spot market.

By trading financial derivatives, strategic producers affect their own and their competitors' strategies, and therefore market competitiveness. We show that each producer uses derivatives to commit to a downward sloping supply function, i.e. to produce more when prices are low and less when they are high. As illustrated in Figure 1 this commitment makes the residual demand curve for each of its competitors steeper (less price-sensitive) and induces them to increase mark-ups and reduce total output. In the aggregate, firms commit to a downward sloping supply function, which increases the volatility of the spot price as even a small demand shock will cause large price fluctuations.

Without derivative trade, producers would offer upward-sloping supply functions in equilibrium.<sup>3</sup> However, producers can influence the slope of their supply function by cleverly using derivatives as commitment devices. It is well-established, that selling a commodity with forward contracts changes a strategic producer's incentives; it becomes more interested in output and less in mark-ups in the spot market.<sup>4</sup> Thus in order to commit to a downward sloping supply function, a firm would want to sell most of its output with contracts when the spot price is low, which commits the firm to a high output, and to sell little of its output with contracts when the spot price is high, which commits the firm to a low output. A producer can achieve this by means of a portfolio of option contracts.

A call option gives the buyer the right to procure one unit of the good from the seller at a predetermined price, the option's strike price. The contract is only exercised when the spot price is above the strike price. Thus a producer can commit to a supply function with a negative slope by first selling a forward contract, which is always exercised, and then buying a portfolio of call options with

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<sup>1</sup>Commodity derivatives markets have seen a 60-fold increase in the value of trade between 1998 and 2008. In 2008 the outstanding value of commodity derivatives equaled \$13 trillion. This is twice the worldwide output of commodities, and about 21% of world GDP.

<sup>2</sup>The effect of derivatives trade is a point of debate in the finance literature. Some authors claim that it reduces the variance and level of spot prices and improves price information (Turnovsky, 1983; Cox, 1976; Korniotis, 2009), while others claim the opposite (Hart and Kreps, 1986; Stein, 1987; Figlewski, 1981).

<sup>3</sup>See Klemperer and Meyer (1989). Vives (2011) shows that there can be exceptions if production costs are private information. Producers might then hold back supply at high prices in order to avoid a winner's curse, and this may result in downward sloping supply functions.

<sup>4</sup>The strategic effect of forward contracts has been shown empirically by Wolak (2000) and Bushnell et al. (2008) and derived theoretically by Allaz and Vila (1993), von der Fehr and Harbord (1992), Newbery (1998) and Green (1999). de Frutos and Fabra (2012) show that there are sometimes exceptions from this rule in markets where offers are required to be stepped.

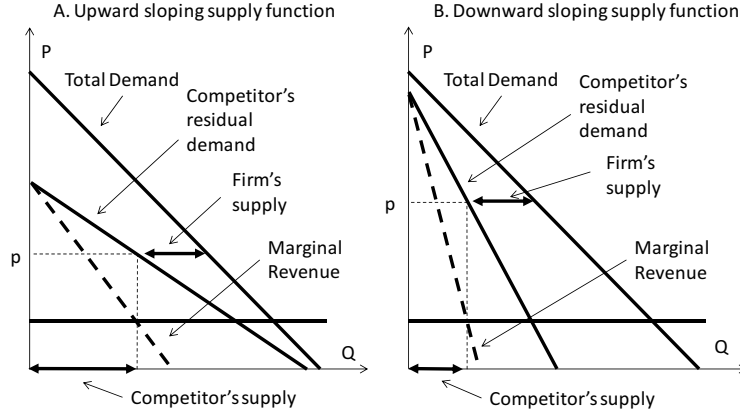


Figure 1: The residual demand function and the production level of a firm's competitor, if the firm bids (A) an upward sloping or (B) a downward sloping supply function.

a range of strike prices, in order to successively reduce its net-sales with exercised contracts as the spot price increases. In other words, the firm chooses a portfolio of contracts, such that the price sensitivity of the portfolio's value decreases with an increasing spot price.

In their seminal paper, Allaz and Vila (1993) analyze a strategic contracting game, where risk-neutral firms first sell forward contracts and then compete with quantities in the spot market. But this set-up is too restrictive when analyzing contracts' influence on the slope of producers' supply. Thus we generalize their two-stage game by allowing for price contingent contracts and supply offers. In the first stage, oligopoly producers choose a portfolio of call option contracts with a spectrum of strike prices. In the second stage, they compete with supply functions in the spot market as in Klemperer and Meyer (1989) and Green and Newbery (1992). Like us Chao and Wilson (2005) consider the influence of option contracts on supply function competition, but contracting is exogenous in their model. As is common in the literature on strategic contracting, this paper focuses on short-run effects and disregards the relation between contracts, entry and investment decisions.<sup>5</sup> We also assume that contract positions are observable<sup>6</sup> and that consumers

<sup>5</sup>Newbery (1998) shows that producers sell contracts to keep output high and spot prices low to deter entry. Murphy and Smeers (2010) show that the impact of forward contracts on competition is ambiguous once investment decisions are endogenized. In Argenton and Willems (2010) firms sell standard forward contracts to exclude potentially more efficient entrants, and Petropoulos et al. (2010) show that financial contracts might lead to preemptive overinvestments by incumbent firms.

<sup>6</sup>Contracting is strategic when firms are risk-averse or when contract positions are observable (Hughes and Kao, 1997). Financial trading is anonymous in most markets, and a firm's contract positions are normally not revealed to competitors. Still competitors can get a rough estimate of the firm's forward position by analyzing the turnover in the forward market and the forward price (Ferreira, 2006). Ferreira's theoretical argument is also relevant in practice. Van Eijkel and Moraga-González's (2010) find that firms in the Dutch gas market are able to infer competitors' contract positions and that contracts are used for strategic rather than hedging reasons. Finally, it can be noted that vertical integration with a retailer that is selling the good at a fixed retail

and financial traders are non-strategic.

Our theoretical predictions are supported by some empirical evidence. First, until recently a handful of electricity producers in the Nordic countries regularly made offers that were (partly) downward sloping.<sup>7</sup> Market observers have attributed those irregular bids to the contractual obligations of those firms, but it has not been formally established to what extent contracts were used intentionally to soften competition in the market. Second, Willems et al. (2009) compare two contracting scenarios for the German electricity market: one with standard forward contracts and another with load following contracts, which corresponds to firms selling forward contracts and several call option contracts.<sup>8</sup> They find that the first scenario fits the data best. This is in line with our results as we predict that firms have no incentive to sell call option contracts, but to buy them instead.

The main results of our paper have parallels in the literature on delegation games. Singh and Vives (1984) and Cheng (1985) analyze a game where the owner of each firm first decides the slope of its supply; it can be either horizontal (Bertrand) or vertical (Cournot). The implementation of this decision is delegated to the firm's manager, which then sets the firm's price or output depending on the owner's chosen slope. In equilibrium, each firm commits to play Cournot when demand is certain. As in our model, this makes the residual demand function of competitors less elastic, it softens competition and leads to higher mark-ups. However, Reisinger and Rössner (2009) show that if demand is sufficiently uncertain, firms commit to play Bertrand. This is similar to our model; we also find that demand uncertainty makes the market more competitive.

The structure of the paper is as follows. Section 2 summarizes the previous literature. The model of strategic option contracting is introduced in Section 3 and its main properties are derived in Section 4. Section 5 presents closed-form results when demand is linear and demand shocks are Pareto distributed of the second-order. Section 6 concludes.

## 2 Literature review

Most oligopoly models with strategic commitments can be analyzed in the framework developed by Fudenberg and Tirole (1984) and Bulow et al. (1985). A firm maximizes its profit by committing to a strategy that softens the response of its competitors. Firms commit to aggressive spot market bidding (high output or low mark-up) if it results in a soft response from competitors (low output or high mark-up), which is, by definition, the case when strategies are substitutes. On the

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price is equivalent to observable contracting.

<sup>7</sup>Downward sloping supply bids were allowed in the Nordic power exchange (Nord Pool) until it introduced a new clearing algorithm on October 10, 2007 that could no longer handle them. Note that the total supply of a firm consists of its supply in the power exchange plus its supply delivered directly to consumers with bilateral contracts. So, even though most power exchanges do not allow for downward sloping supply functions, total supply might still be downward sloping if bilaterally contracted supply is sufficiently downward sloping.

<sup>8</sup>With the load following contracts, the firm has sold a set of call option contracts such that for each price level in the spot market, the same fraction of output is hedged.



other hand, firms commit to peaceable (soft) spot market bidding if this results in a soft response from competitors.

The seminal paper on strategic forward contracting by Allaz and Vila (1993) is an example of a game with strategic substitutes. They analyze a two-stage Cournot model with a homogeneous good. The set-up introduces a prisoners' dilemma for producers: Each producer will sell forward contracts to commit to large output. This gives it a Stackelberg first-mover advantage, which would reduce competitors' output, if they would not have sold any forward contracts themselves. However, in equilibrium all firms sell forward contracts, spot market competition becomes tougher, and producers end up worse off. Thus endogenous contracting is pro-competitive and welfare increasing when firms compete with quantities, which Brandts et al. (2008) confirm in economic experiments.<sup>9</sup>

However, introducing a forward market worsens competition when strategies are complements, i.e. when an aggressive commitment results in an aggressive spot market response from competitors. Mahenc and Salanié (2004) analyze a market with differentiated goods and price competition, and show that a commitment to low mark-ups, due to forward sales, is met with a tough response, that is competitors also lower their mark-ups. To avoid the tough response, firms buy in the forward market (negative contracting) in order to soften competition in the spot market. This increases mark-ups in the spot-market. Thus, forward trading reduces social welfare when strategies are complements.

In a more generalized form of spot market competition, producers compete with supply functions, as in the supply function equilibrium (SFE) model (Klemperer and Meyer, 1989). The setting of the SFE model is obviously well-suited for markets where producers sell their output in a uniform-price auction, as in wholesale electricity markets (Green and Newbery, 1992; Holmberg and Newbery, 2010).<sup>10</sup> Although most markets are not explicitly cleared by uniform-price auctions, Klemperer and Meyer (1989) argue that firms typically face a uniform market price and they need predetermined decision rules for its lower-level managers on how to deal with changing market conditions. Thus firms implicitly commit to supply functions also in the general case. Indeed, Vives (2011) notes that competition in supply functions has been used to model bidding for government procurement contracts, management consulting, airline pricing reservation systems, and provides a reduced form in strategic agency and trade policy models.

Bertrand and Cournot competition can be seen as two extreme forms of supply function competition. Thus, it is not surprising that the competitive effects of strategic forward contracting are ambiguous in supply function markets. Newbery (1998) considers cases where the demand uncertainty is bounded, so that the market has multiple equilibria. He shows that contracting depends on how firms coordinate their strategies in the spot market. Green (1999) shows that forward contracting strategies are neutral (neither pro- nor anti-competitive) in markets

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<sup>9</sup>Allaz and Vila (1993) also show that the perfectly competitive outcome is a subgame perfect Nash equilibrium if contracting is repeated an infinite number of rounds. However, Ferreira (2003) proves that this outcome is not renegotiation-proof, while the monopolistic outcome is.

<sup>10</sup>Empirical studies (Hortacsu and Puller, 2008; Sioshansi and Oren, 2007; Wolak, 2003) verify that large producers in the electricity market roughly bid as predicted by the SFE model.

with linear marginal costs and linear demand if producers coordinate on linear supply function equilibria. Holmberg (2011) considers a sufficiently wide support of the demand shock distribution, so that the spot market has a unique equilibrium. He shows that contracting strategies are substitutes (pro-competitive) when marginal costs are convex and residual demand is concave and that the reverse is true when marginal costs are concave and residual demand is convex. Holmberg (2011) also shows that forward trading is weakly pro-competitive when marginal costs are constant. Herrera-Dappe (2008) calculates asymmetric contracting equilibria and in his setting forward trading will decrease welfare.<sup>11</sup>

In our setting firms trade option contracts, so the contracted quantity is contingent on the spot price. This is the main difference in our study compared to the above mentioned studies. Similarly, Willems (2005) generalizes Allaz and Vila's (1993) result by considering firms that sell a bundle of option contracts, but Willems keeps the Cournot setting in the spot market. As in Allaz and Vila (1993) a firm can commit to be tough by selling more contracts. However strategies are stronger strategic substitutes if financial option contracts are used. By being tough a firm lowers the spot price, which reduces the number of option contracts that are exercised by competitors.<sup>12</sup> This softens the competitors' actions more than with forward contracts. Thus the incentive to sell contracts increases, and the equilibrium becomes more competitive. Selling physical call options has an additional effect; a firm can commit to a positively sloped supply function, even if it is restricted to playing Cournot in the spot market. As this makes the competitors' residual demand more elastic, it toughens their response. Actions become weaker strategic substitutes or even complements. Hence, firms will commit less or nothing at all. Therefore the market becomes less competitive with physical options compared to financial options in Willems' Cournot model. In our current paper we show that in a supply function model, the physical and the financial option contracts are equivalent, as both contracts affect the slope of the offer functions in the second stage. We also show that firms prefer their contracted quantity to decrease with respect to the spot price, so that the slope of competitors' residual demand becomes less elastic. Thus, unlike Willems (2005) firms buy call option contracts in our model.

### 3 Model

We model producers' contracting and supply strategies as a two-stage game. In the first stage,  $N$  risk-neutral producers commit by strategically choosing a portfolio of call option contracts with a spectrum of strike prices. Firms' contracting deci-

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<sup>11</sup>Anderson and Xu (2005; 2006), Anderson and Hu (2012), Aromi (2007), Chao and Wilson (2005) and Niu et al. (2005) have also analyzed how exogenously given forward or option contracts influence supply function competition. But they do not analyze to what extent contracting is strategically driven.

<sup>12</sup>A similar pro-competitive mechanism occurs when each firm offers its forward contracts with a supply function in the contract market (Green, 1999; Holmberg, 2011). Then a firm has incentives to sell forward contracts in order to reduce the forward price and competitors' forward sales.

sions are made simultaneously. Similar to Allaz and Vila (1993), Newbery (1998), Green (1999) etc., producers disclose their contracting decisions. Risk-neutral, non-strategic counterparties (e.g. consumers or investment banks) with rational expectations ensure that each option price corresponds to the expected value of the contract. This rules out any arbitrage opportunities in the market. In the second stage, firms compete in the spot market. It is a uniform-price auction in which sellers simultaneously submit supply functions. After these offers have been submitted, an additive demand shock is realized. The distribution of the shock is common knowledge.

A financial call option gives the buyer the right to receive the difference between the spot market price and a predetermined strike price  $r$ .<sup>13</sup> The contract is exercised when the spot price is above the strike price. In stage 1, firm  $i \in \{1, 2, \dots, N\}$  decides how many option contracts to sell at each strike price. We assume that 0 and  $\bar{p}$  are the lowest and highest realized prices in the market, respectively. The contracting decision is represented by the right-continuous distribution function  $X_i(r) : [0, \bar{p}] \rightarrow \mathbb{R}$ , the amount of option contracts sold with a strike price equal to or below  $r$ . Firm  $i$  can decide to go short ( $X_i(r) > 0$ ) or long ( $X_i(r) < 0$ ). The sum of competitors' contracting decisions is given by  $X_{-i}(r) = \sum_{j \neq i} X_j(r)$ .

Let  $\sigma(r)$  be the price of an option with strike price  $r$  in the contracting market. A forward contract corresponds to a call option with strike price zero, i.e. it is always exercised. Thus  $X_i(0)$  is the amount of sold forward contracts and  $\sigma(0)$  is the forward price. A producer's revenue from sales of call options in the contracting market is given by:<sup>14</sup>

$$\bar{V}_i = \int_0^{\bar{p}} \sigma(r) \cdot dX_i(r).$$

Each producer's supply decision in stage 2, the spot market, is represented by a right-continuous supply function denoted by  $Q_i(p)$ . We let  $Q_{-i}(p) := \sum_{j \neq i} Q_j(p)$  be the aggregate supply function of all firms, except firm  $i$ . We also introduce the net-supply function  $S_i(p) := Q_i(p) - X_i(p)$  and we let  $S_{-i}(p) = \sum_{j \neq i} S_j(p)$ . We assume that  $Q_i(p)$  and  $X_i(p)$  are twice differentiable on  $(0, \bar{p})$  and continuous at  $\bar{p}$ .

As in Klemperer and Meyer (1989), demand  $D(p, \varepsilon)$  is realized after offers to the spot market have been submitted. The demand function is twice differentiable with respect to the spot price  $p$  and is subject to an exogenous additive shock,  $\varepsilon$ . Hence,

$$D(p, \varepsilon) = D(p) + \varepsilon. \quad (1)$$

The demand function is concave ( $D''(p) \leq 0$ ), downward sloping ( $D'(0) < 0$ ) and satisfies  $D(0) = 0$ . When firms make their contracting decisions in the first stage, the shock density and its probability distribution are given by  $f(\varepsilon)$  and  $F(\varepsilon)$ , respectively. The shock density has support on  $[0, \bar{\varepsilon}]$  and on this interval  $f(\varepsilon)$  is differentiable and positive,  $f(\varepsilon) > 0$ . The variance of the demand shock is bounded.

<sup>13</sup>We show that the results are identical under physical contracts.

<sup>14</sup>Note that we use the Lebesgue-Stieltjes integral, which is standard in probability theory.

The market clearing price in the spot market is for any given demand shock,  $\varepsilon$ , defined implicitly by the market clearing condition: aggregate supply should be equal to total demand. The price function  $P(\varepsilon)$  maps the demand shock  $\varepsilon$ , to the market equilibrium price  $p$ .

$$P(\varepsilon) : \varepsilon \mapsto p : \sum_{i=1}^N Q_i(p) = D(p) + \varepsilon.$$

To guarantee existence of an equilibrium price, we assume as in Klemperer and Meyer (1989) that all agents' profits will be zero if the market does not clear.

A producer's revenue from selling in the spot market is given by:

$$P(\varepsilon) \cdot Q_i(P(\varepsilon)).$$

All call options that are in the money will be exercised. Thus for a shock outcome  $\varepsilon$ , the total value  $V_i(P(\varepsilon))$  of firm  $i$ 's sold contracts is given by: <sup>15</sup>

$$V_i(P(\varepsilon)) = \int_0^{P(\varepsilon)} (P(\varepsilon) - r) dX_i(r) = \int_0^{P(\varepsilon)} X_i(r) dr.$$

Note that the sensitivity of this contract payment with respect to the spot price is exactly equal to  $X_i(r)$

$$\frac{dV_i(p)}{dp} = X_i(p). \quad (2)$$

Firm  $i$  has a cost function,  $C_i(\cdot)$ . Similar to Green and Newbery (1992), and Klemperer and Meyer (1989) we assume that it is common knowledge, increasing, convex and twice differentiable. The total production cost for firm  $i$  when shock  $\varepsilon$  is realized is given by

$$C_i(Q_i(P(\varepsilon))).$$

Firm  $i$ 's profit from trading in the contract and the spot markets is equal to the revenue from sold contracts and spot market sales, minus the cost of exercised contracts and the production cost. For a given spot price  $p$  and output  $q$ , the profit of firm  $i$  is equal to <sup>16</sup>

$$\pi_i(p, q) = \bar{V}_i + pq - V_i(p) - C_i(q). \quad (3)$$

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<sup>15</sup>Note that  $P(\varepsilon) - r$  is continuous in  $r$ . Thus this follows from the integration by parts formula for the Lebesgue-Stieltjes integral, where one of the factors is continuous at each point (Hewitt and Stromberg, 1965).

<sup>16</sup>Note that the pay-off is identical for physical contracts.

$$\begin{aligned} \pi_i(p, q) &= \bar{V}_i + \underbrace{\int_0^p r dX_i(r)}_{\text{Revenue from exercised contracts}} + \underbrace{p(q - X(p))}_{\text{Spot revenue}} - C_i(q) \\ &= \bar{V}_i + \int_0^p (r - p) dX_i(r) + pq - C_i(q) \\ &= \bar{V}_i - V_i + pq - C_i(q). \end{aligned}$$

Thus, in stage 1, firm  $i$ 's expected profit from trading in the contract and the spot markets is:

$$\begin{aligned} \Pi_i &= E_\varepsilon [\pi_i(P(\varepsilon), Q_i(P(\varepsilon)))] \\ &= \bar{V}_i + E_\varepsilon \left[ P(\varepsilon) \cdot Q_i(P(\varepsilon)) - V_i(P(\varepsilon)) - C_i(Q_i(P(\varepsilon))) \right]. \end{aligned} \quad (4)$$

Risk-neutral, price-taking consumers or investment banks trade in the contract market and ensure that the following non-arbitrage condition is satisfied for each strike price  $r$ .

$$\forall r : \quad \sigma(r) = E_\varepsilon [\max(P(\varepsilon) - r, 0)]. \quad (5)$$

Hence, the value of the call option is equal to the expected second stage payment from the contract.

## 4 Analysis

We solve the game by means of backward induction. The properties of Nash equilibria in the second stage spot market are analyzed in Section 4.1. In Section 4.2, we rely on non-arbitrage conditions to derive the expected profit in stage 1 given the contracting position of firms. Our equilibrium concept for the two stage game is discussed in detail in Section 4.3, before we derive conditions for optimal contracting in Section 4.4.

### 4.1 The spot market

In the second stage of the game, each firm  $i$  observes its competitors' portfolio of option contracts and then chooses its supply function  $Q_i(p)$  to maximize the firm's expected profit given the competitors' spot market bids  $Q_{-i}(p)$ .

Due to the demand shock, firm  $i$  faces an unknown residual demand curve in the spot market. As in Anderson and Philpott (2002a) and Wilson (1979), we use the market distribution function  $\psi_i(p, q)$  to characterize residual demand. The function returns the probability that a supply offer  $q$  at price  $p$  from firm  $i$  is rejected. In our setting this implies that

$$\psi_i(p, q) = F(\underbrace{Q_{-i}(p) + q - D(p)}_\varepsilon). \quad (6)$$

Firm  $i$  chooses its bid function  $Q_i(p)$  to maximize its expected profit which can be written as the line integral (Anderson and Philpott, 2002a):

$$\max_{Q_i(p)} \int_{Q_i(p)} \pi_i(p, q) d\psi_i(p, q). \quad (7)$$

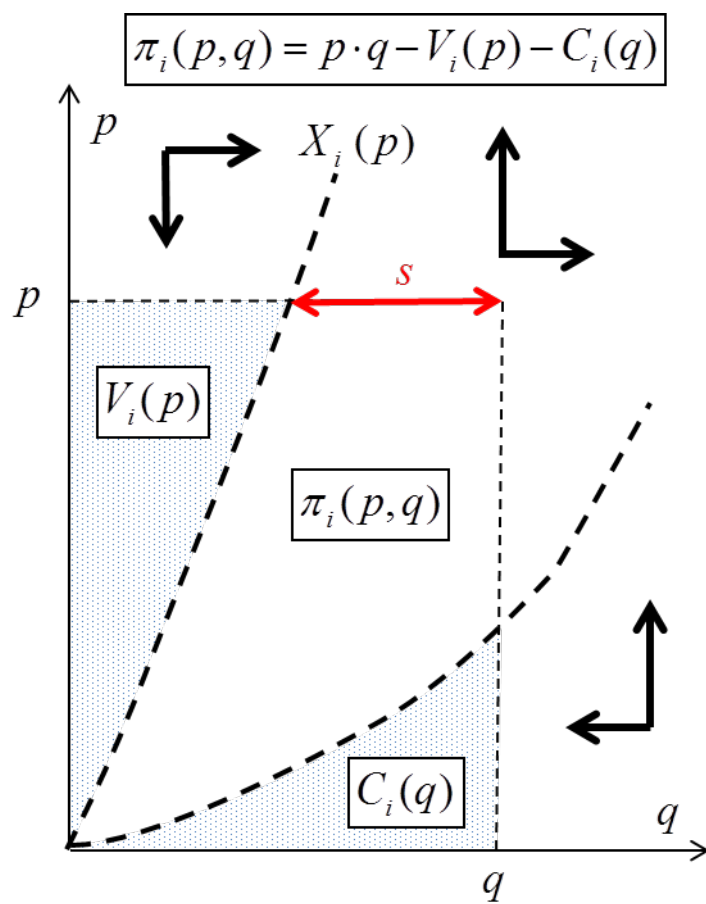


Figure 2: Profit of firm  $i$  as a function of  $p$  and  $q$ .

**Proposition 1** *The necessary first-order condition of program (7) is that along the optimal bid function  $Q_i(p)$  the market distribution function and the iso-profit function of firm  $i$  are tangent as illustrated in Figure 3, i.e.  $\forall(q, p)$  such that  $q = Q_i(p)$  and  $p \in (0, \bar{p})$ ,<sup>17</sup>*

$$\frac{\partial \pi_i}{\partial q} \cdot \frac{\partial \psi_i}{\partial p} - \frac{\partial \pi_i}{\partial p} \cdot \frac{\partial \psi_i}{\partial q} = 0. \quad (8)$$

**Proof.** We rewrite equation (7) as

$$\max_{Q(p)} \int_0^{\bar{p}} \underbrace{\pi_i(p, Q(p)) \cdot \left[ \frac{\partial \psi_i}{\partial q}(p, Q(p)) \cdot Q'(p) + \frac{\partial \psi_i}{\partial p}(p, Q(p)) \right]}_{L(p, Q(p), Q'(p))} dp$$

and then use the Euler equation  $\frac{\partial L}{\partial Q} - \frac{d}{dp} \left( \frac{\partial L}{\partial Q'} \right) = 0$  to derive the necessary first order condition. Expanding the partial derivatives of  $L$

$$\begin{aligned} \frac{\partial L}{\partial Q} &= Q' \left[ \frac{\partial \pi_i}{\partial q} \frac{\partial \psi_i}{\partial q} + \pi_i \frac{\partial^2 \psi_i}{\partial q^2} \right] + \left[ \frac{\partial \pi_i}{\partial q} \frac{\partial \psi_i}{\partial p} + \pi_i \frac{\partial^2 \psi_i}{\partial q \partial p} \right] \\ \frac{d}{dp} \left( \frac{\partial L}{\partial Q'} \right) &= Q' \left[ \frac{\partial \pi_i}{\partial q} \frac{\partial \psi_i}{\partial q} + \pi_i \frac{\partial^2 \psi_i}{\partial q^2} \right] + \left[ \frac{\partial \pi_i}{\partial p} \frac{\partial \psi_i}{\partial q} + \pi_i \frac{\partial^2 \psi_i}{\partial q \partial p} \right] \end{aligned}$$

and simplifying the Euler equation proves the proposition. ■

We are now ready to derive a generalized version of the first-order condition in Klemperer and Meyer (1989) that considers contracts:

**Proposition 2** *[FOC Spot Market] The necessary first-order conditions (FOC) for any Nash equilibrium in the spot market are given by the following system of ordinary differential equations:*

$$\forall i, \forall p \in (0, \bar{p}) : S_i(p) + [p - C'_i(Q_i(p))] [D'(p) - Q'_{-i}(p)] = 0. \quad (9)$$

**Proof.** From (2), (3) and (6) we can calculate the partial derivatives of  $\psi_i$  and  $\pi_i$ :

$$\frac{\partial \psi_i}{\partial q} = f(Q_{-i}(p) + q - D(p)) \quad (10)$$

$$\frac{\partial \psi_i}{\partial p} = [Q'_{-i}(p) - D'(p)] \cdot \frac{\partial \psi_i}{\partial q} \quad (11)$$

$$\frac{\partial \pi_i}{\partial q} = p - C'_i(q) \quad (12)$$

$$\frac{\partial \pi_i}{\partial p} = q - X_i(p) \quad (13)$$

which we can substitute into (8) to derive equation (9). ■

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<sup>17</sup>This is a similar first-order condition as derived for uncontracted firms by Anderson and Philpott (2002a).

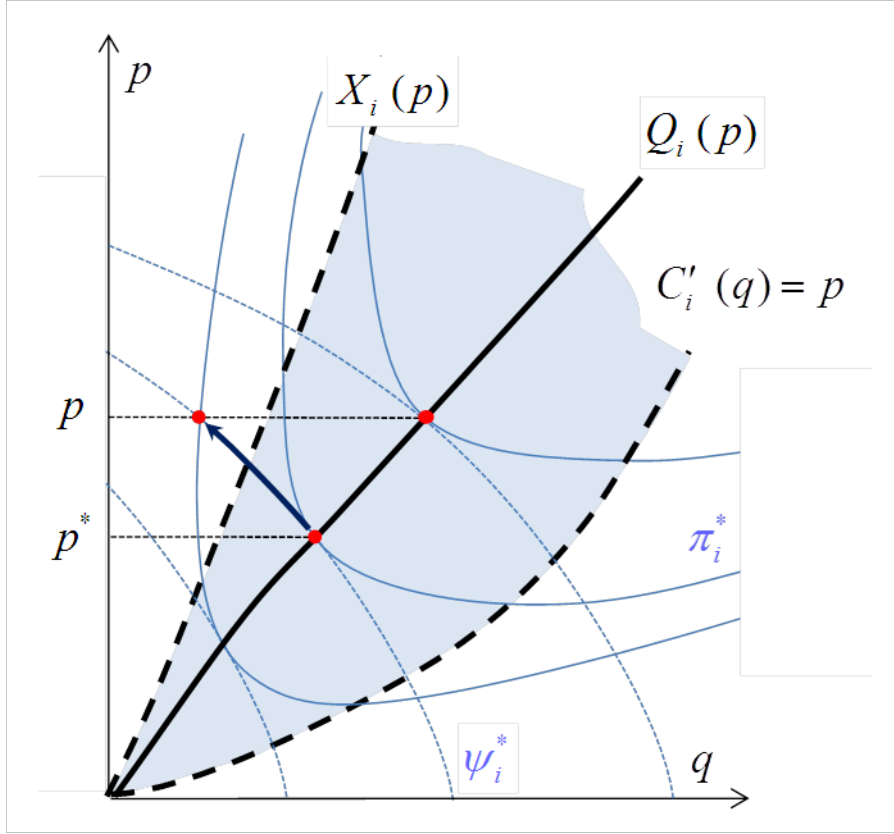


Figure 3: The first order condition requires that iso-profit lines and market distribution contours are tangent along the optimal supply function.

Note that in our setting, the demand shock is additive; it shifts the residual demand curve of firm  $i$  horizontally and  $\psi_i$  is constant along each realized residual demand curve. Hence, the optimal net-supply curve  $S_i(p)$  is also *ex-post optimal* as it crosses each residual demand curve at a point where the latter is tangent to the firm's iso-profit line as in Figure 3. This implies that the probability distribution of the shock has no influence on optimal bidding in the spot market. Each producer simply acts as a monopolist with respect to its residual demand curve (net of option contracts) for each shock outcome and sets a price according to an inverse elasticity rule (Tirole, 1988)

$$\frac{p - C'_i(Q_i)}{p} = \frac{-S_i/p}{D'(p) - Q'_{-i}(p)} = -\frac{1}{\varepsilon_{\text{Residual Demand}}}.$$

This equation can be found by rewriting the generalized Klemperer-Meyer condition (9).

The following defines a well-behaved solution to the system of first order conditions (FOC).

**Definition 1** [Well-behaved FOC solution] The tuple  $\check{\mathbf{S}} = \{\check{S}_i(p)\}_{i=1}^N$  is a well-behaved FOC solution of the second stage game, if



1. The supply functions solve the first order conditions of the spot market, i.e. the generalized Klemperer and Meyer equations (9)
2. The slope of total supply is larger than the slope of the demand function  $\sum_{j=1}^N \check{Q}'_j(p) > D'(p)$  on the price interval  $(0, \bar{p})$ . This ensures that clearing prices are unique.
3. Each firm  $i$  faces a downward sloping residual demand function or has sufficiently flat marginal cost functions. That is for  $\forall q \geq 0$  and  $\forall p \in (0, \bar{p})$  it must be that  $\left[ D'(p) - \sum_{j \neq i}^N \check{Q}'_j(p) \right] C''_i(q) \leq 1$ .

**Proposition 3** [NE Spot Market]<sup>18</sup> The well-behaved FOC solution  $\check{\mathbf{S}}$  constitutes a Nash equilibrium (NE) in the second-stage.

**Proof.** Consider an arbitrary firm  $i$ . It takes contract positions  $\{X_i(p)\}_{i=1}^N$  as given and assumes that its competitors bid  $\check{Q}_{-i}(p)$  as supply. We prove that bidding  $\check{Q}_i(p)$  is profit maximizing for firm  $i$ .

Let  $D_i^*(p) = D(p) + \varepsilon^* - \sum_{j \neq i}^N \check{Q}_j(p)$  be the residual demand curve of firm  $i$  for the shock  $\varepsilon^*$ . When demand shock  $\varepsilon^*$  occurs, the equilibrium price  $p^*$  is determined by  $\check{Q}_i(p^*) = D_i^*(p^*)$   $\square$ . We will prove that firm  $i$ 's profit reaches a global maximum at  $p^*$  along its residual demand  $D_i^*(p)$  for every shock outcome  $\varepsilon^*$ . That is, producing  $\check{Q}_i(p^*)$  is ex-post optimal for firm  $i$ .

We analyze how firm  $i$ 's profits change along the residual demand function  $D_i^*(p)$ . It follows from (12) and (13) that

$$\begin{aligned} \frac{d\pi_i(p, D_i^*(p))}{dp} &= \frac{\partial \pi_i}{\partial p} + D_i^{*'} \frac{\partial \pi_i}{\partial q} \\ &= [D_i^*(p) - X_i(p)] + D_i^{*'} [p - C'_i(D_i^*(p))] . \end{aligned}$$

Subtracting the first-order condition in (9) from this expression yields:

$$\frac{d\pi_i(p, D_i^*(p))}{dp} = [D_i^*(p) - \check{Q}_i(p)] - D_i^{*'}(p) [C'_i(D_i^*(p)) - C'_i(\check{Q}_i(p))] . \quad (14)$$

According to the mean value theorem there must exist  $\xi \in [D_i^*(p), \check{Q}_i(p)]$  such that  $C'_i(D_i^*(p)) - C'_i(\check{Q}_i(p)) = C''_i(\xi) [D_i^*(p) - \check{Q}_i(p)]$ . So Equation 14 can be rewritten as:

$$\frac{d\pi_i(p, D_i^*(p))}{dp} = [D_i^*(p) - \check{Q}_i(p)] (1 - D_i^{*'}(p) C''_i(\xi))$$

Definition 1.3 implies that  $D_i^{*'}(p) \cdot C''_i(\xi) \leq 1$ . So the second factor of the expression is always positive. The market clears at price  $p^*$ , so the first factor is zero when  $p = p^*$ . Definition 1.2 implies that for all price levels  $\check{Q}'_i(p) > D'_i(p)$  so the first

<sup>18</sup>This proposition generalizes a result by Holmberg et al. (2008) for spot markets without contracting.

factor is negative when prices  $p$  are above  $p^*$  and positive for prices below  $p^*$ . Hence we have shown that:

$$\frac{d\pi_i(p, D_i^*(p))}{dp} \begin{cases} \leq 0 & \text{if } p > p^* \\ = 0 & \text{if } p = p^* \\ \geq 0 & \text{if } p < p^* \end{cases}$$

which is sufficient for a global profit maximum at  $p^*$ . We can use the same argument for all shocks  $\varepsilon$  and all firms  $i$  and we can conclude that the well-behaved FOC solution is a Nash equilibrium. ■

## 4.2 First Period Profit function under perfect arbitrage

The no-arbitrage condition (5) is valid for any contracting choice of the producers. By using the arbitrage condition and reversing the order of integration, we can rewrite the contracting revenue of firm  $i$ :

$$\begin{aligned} \bar{V}_i &= \int_0^{\bar{p}} \sigma(r) \cdot dX_i(r) \\ &= \int_0^{\bar{p}} E_\varepsilon [\max(P(\varepsilon) - r, 0)] \cdot dX_i(r) \\ &= E_\varepsilon [V_i(P(\varepsilon))]. \end{aligned} \tag{15}$$

Thus due to perfect arbitrage, the contracting revenue is equal to the expected realized value of the portfolio. Substituting the expected contract revenue (15) into the pay-off (4) and simplifying we obtain:

$$\Pi_i = E_\varepsilon [\pi_i(\varepsilon)] = E_\varepsilon [P(\varepsilon) \cdot Q_i(P(\varepsilon)) - C_i(Q_i(P(\varepsilon)))]. \tag{16}$$

Thus firm  $i$ 's pay-off does not depend on the contract position directly, but by selling contracts ( $X_i$ ), it can strategically change the price in the spot market  $P(\varepsilon)$ .

## 4.3 Equilibrium concept

Klemperer and Meyer (1989) show that multiple Nash equilibria may exist in supply function games such as the one played in the second stage of our model. This complicates the equilibrium analysis of our game. Before proceeding we therefore discuss how we refine the sub-game perfect Nash equilibrium concept, and which solution strategy we will follow to find such an equilibrium. We start with some definitions.

Each tuple of contracting actions  $\mathbf{X} = \{X_i(p)\}_{i=1}^N$  defines a different subgame in the second stage spot market, which we denote as *subgame*  $\mathbf{X}$ . The set of subgames will be denoted  $\Xi$ . Firm  $i$ 's *strategy*  $\{X_i(p), Q_i(p, \mathbf{X})\}$  specifies its action in the first stage (the contract market) and in each second stage subgame (the spot market). The *strategy profile*, the set of all firms' strategies, is given by  $\{\mathbf{X}, \mathbf{Q}(\mathbf{X})\}$ ,

where  $\mathbf{Q}(\mathbf{X}) = \{Q_i(p, \mathbf{X})\}_{i=1}^N$  specifies a tuple of supply functions for each subgame  $\mathbf{X}$ . An *outcome* is a set of actions that firms take along one particular path in the game. Hence we present an outcome as  $\{\mathbf{X}^0, \mathbf{Q}^0\}$ ; the contract curves  $\mathbf{X}^0 = \{X_i(p)\}_{i=1}^N$  and supply functions  $\mathbf{Q}^0 = \{Q_i(p)\}_{i=1}^N$  that firms offer in the second stage. We let  $SFE(\mathbf{X})$  be the *set of Supply Function Equilibria in subgame  $\mathbf{X}$* , and  $FOC(\mathbf{X})$  the set of tuples  $\mathbf{Q}(\mathbf{X})$  in subgame  $\mathbf{X}$  that satisfy the necessary first order conditions of an SFE as specified in Proposition 2. Thus  $SFE(\mathbf{X}) \subseteq FOC(\mathbf{X})$ .

To rule out non-credible threats in the subgames, we solve for Subgame Perfect Nash Equilibrium (SPNE) in the two-stage game.

**Definition 2** [SPNE] A strategy profile  $\{\mathbf{X}^*, \mathbf{Q}^*(\mathbf{X})\}$  constitutes a Subgame Perfect Nash Equilibrium (SPNE) iff:

$$\begin{aligned} (i) \quad & \forall i, \forall X_i \quad \Pi_i(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*) \geq \Pi_i(\mathbf{Q}^*(\{X_i, \mathbf{X}_{-i}^*\}), \{X_i, \mathbf{X}_{-i}^*\}) \\ (ii) \quad & \forall \mathbf{X} \in \Xi \quad \mathbf{Q}^*(\mathbf{X}) \in SFE(\mathbf{X}). \end{aligned}$$

The first equation specifies that firms do not have an incentive to change their contracting decisions in the first stage, given competitors' contracting strategies  $\mathbf{X}_{-i}^*$  and the equilibria which will be played in the second stage, as described by  $\mathbf{Q}^*(\mathbf{X})$ . The second equation states that the function  $\mathbf{Q}^*(\mathbf{X})$  has to be an equilibrium in each subgame.

There can be multiple SPNE when subgames do not have unique equilibria. In this case, we use Pareto dominance to refine the set of Nash equilibria in subgames with multiple Nash Equilibria.

**Definition 3** [Pareto Optimality] A Supply Function Equilibrium  $\mathbf{Q}^* \in SFE(\mathbf{X})$  in subgame  $\mathbf{X}$  is Pareto optimal if no alternative equilibrium  $\mathbf{Q} \in SFE(\mathbf{X})$  exists which is weakly preferred by all firms  $\Pi_i(\mathbf{Q}, \mathbf{X}) - \Pi_i(\mathbf{Q}^*, \mathbf{X}) \geq 0$ , and strictly by at least one firm  $j$ :  $\pi_j(\mathbf{Q}, \mathbf{X}) - \pi_j(\mathbf{Q}^*, \mathbf{X}) > 0$ .

We let  $SFE_{PO}(\mathbf{X})$  be the set of Pareto optimal SFE in subgame  $\mathbf{X}$ .

**Definition 4** [PO-SPNE] A subgame Perfect Nash Equilibrium (SPNE)  $\{\mathbf{X}^*, \mathbf{Q}^*(\mathbf{X})\}$  is a Pareto Optimal subgame Perfect Nash Equilibrium (PO-SPNE) if:<sup>19</sup>

$$\forall \mathbf{X} \in \Xi \quad \mathbf{Q}^*(\mathbf{X}) \in SFE_{PO}(\mathbf{X}).$$

This implies that the equilibria are renegotiation proof in each subgame, and that firms cannot rely on grim punishments strategies off the equilibrium path to attain more cooperation. A problem with the PO-SPNE concept is that it can only be applied to games where Pareto Optimal NE exists in every subgame. Although the set  $SFE(\mathbf{X})$  is typically non-empty, it might not be closed, so  $SFE_{PO}(\mathbf{X})$  might be an empty set for some subgames. We therefore introduce the following generalization of Pareto optimality and the PO-SPNE:

<sup>19</sup>The Pareto Perfect Equilibrium (PPE) (Bernheim et al., 1987) is related with the PO-SPNE concept, but is a stricter refinement as it imposes that only those PO-SPNE are played which are Pareto Optimal in the first stage. Hence, the set of PPE is a subset of the set of PO-SPNE. In case a PO-SPNE is unique, then it is also a Pareto Perfect Equilibrium (PPE).

**Definition 5** [ $\epsilon$ -Pareto Optimality] A Supply Function Equilibrium  $\mathbf{Q}^* \in SFE(\mathbf{X})$  in subgame  $\mathbf{X}$  is  $\epsilon$ -Pareto optimal if no alternative equilibrium  $\mathbf{Q} \in SFE(\mathbf{X})$  exists which is weakly preferred by all firms  $\Pi_i(\mathbf{Q}, \mathbf{X}) - \Pi_i(\mathbf{Q}^*, \mathbf{X}) \geq 0$ , and where one firm  $j$  would gain at least  $\epsilon$ ,  $\pi_j(\mathbf{Q}, \mathbf{X}) - \pi_j(\mathbf{Q}^*, \mathbf{X}) > \epsilon$ .

For a given  $\epsilon \geq 0$ , we define  $SFE_{\epsilon-PO}(\mathbf{X})$  as the set of  $\epsilon$ -Pareto Optimal SFE in subgame  $\mathbf{X}$ . Note that  $SFE_{PO}(\mathbf{X}) \subseteq SFE_{\epsilon-PO}(\mathbf{X}) \subseteq SFE(\mathbf{X})$ .

**Definition 6** [ $\epsilon$ -PO-SPNE] For a given  $\epsilon \geq 0$ , a subgame perfect Nash Equilibrium (SPNE)  $\{\mathbf{X}^*, \mathbf{Q}^*(\mathbf{X})\}$  is an  $\epsilon$ -Pareto Optimal subgame perfect Nash Equilibrium ( $\epsilon$ -PO-SPNE) if:

$$\forall \mathbf{X} \in \Xi \quad \begin{aligned} \mathbf{Q}^*(\mathbf{X}) &\in SFE_{\epsilon-PO}(\mathbf{X}) \\ \mathbf{Q}^*(\mathbf{X}^*) &\in SFE_{PO}(\mathbf{X}^*) \end{aligned}$$

This definition requires that firms play a Pareto Optimal Nash equilibrium along the equilibrium path, while off the equilibrium path, they are allowed to play any Nash equilibrium which is not  $\epsilon$ -dominated by other Nash equilibria in the subgame. So by coordinating on another Nash equilibrium in a subgame off the equilibrium path, no firm can gain more than  $\epsilon$  without making some other firm worse off.

We now introduce two concepts which are often used in operations research to compute equilibria in dynamic games. A Mathematical Program with Equilibrium Constraints (MPEC, Luo et al., 1996) is a optimization program where a firm maximizes its profit, subject to a set of first order conditions (the equilibrium constraints).

**Definition 7** [MPEC Outcome] An outcome  $\{\mathbf{X}^*, \mathbf{Q}^*\}$  is an MPEC outcome for firm  $i$  iff it is a solution of firm  $i$ 's Mathematical Program with Equilibrium constraints (MPEC)

$$MPEC(i) \quad \forall X_i, \forall \mathbf{Q} \in FOC(\{X_i, \mathbf{X}_{-i}^*\}) \quad \Pi_i(\mathbf{Q}^*, \mathbf{X}^*) \geq \Pi_i(\mathbf{Q}, \{X_i, \mathbf{X}_{-i}^*\}).$$

An Equilibrium Program with Equilibrium Constraints (EPEC) is the system of MPECs, one for each firm  $i$ .<sup>20</sup>

**Definition 8** [EPEC Equilibrium] An outcome  $\{\mathbf{X}^*, \mathbf{Q}^*\}$  is an EPEC outcome iff  $\forall i$ ,  $\{\mathbf{X}^*, \mathbf{Q}^*\}$  is a MPEC outcome for firm  $i$ . We say that  $\{\mathbf{X}^*, \mathbf{Q}^*\}$  is an EPEC equilibrium if in addition  $\mathbf{Q}^* \in SFE(\mathbf{X}^*)$ .

It can be shown that the EPEC equilibrium is closely related to the concept of an  $\epsilon$ -PO-SPNE of a dynamic game:

**Proposition 4** If there exists an SFE in every sub game, i.e.  $\forall \mathbf{X} : SFE(\mathbf{X}) \neq \emptyset$ , and the monopoly profit is bounded in every subgame, then an EPEC equilibrium  $\{\mathbf{X}^*, \mathbf{Q}^*\}$  is on the equilibrium path of an  $\epsilon$ -PO-SPNE for any strictly positive value of  $\epsilon$ .

<sup>20</sup>Su (2007) and Zhang et al. (2010) use an EPEC to calculate strategic forward contracting in oligopoly markets.

**Proof. Step 1:** We first prove that the EPEC equilibrium implies that a Pareto Optimal SFE is played along the equilibrium path, i.e.  $\mathbf{Q}^*(\mathbf{X}^*) \in SFE_{PO}(\mathbf{X}^*)$ . We will use a proof by contradiction. Suppose that there is a Nash equilibrium  $\tilde{\mathbf{Q}} \in SFE(\mathbf{X}^*)$  such that

$$\forall i, \Pi_i(\tilde{\mathbf{Q}}, \mathbf{X}^*) \geq \Pi_i(\mathbf{Q}^*, \mathbf{X}^*)$$

where the inequality is strict for one of the firms. Without loss of generality assume that for firm 1:  $\Pi_1(\tilde{\mathbf{Q}}, \mathbf{X}^*) > \Pi_1(\mathbf{Q}^*, \mathbf{X}^*)$ . The definition of the EPEC outcome requires that:

$$\Pi_1(\mathbf{Q}^*, \mathbf{X}^*) \geq \Pi_1(\mathbf{Q}, \mathbf{X}^*) \quad \forall \mathbf{Q} \in FOC(\mathbf{X}^*).$$

As the first order conditions are necessary conditions for an equilibrium, we must have that  $\tilde{\mathbf{Q}} \in FOC(\mathbf{X}^*)$ . Hence,

$$\Pi_1(\mathbf{Q}^*, \mathbf{X}^*) \geq \Pi_1(\tilde{\mathbf{Q}}, \mathbf{X}^*),$$

which is a contradiction.

**Step 2:** We will now prove that in every subgame  $\mathbf{X}$ , and for each  $\epsilon > 0$  there exists an  $\epsilon$ -PO-SFE, i.e.  $\forall \mathbf{X}, \forall \epsilon > 0, SFE_{\epsilon-PO}(\mathbf{X}) \neq \emptyset$ . Define total industry profit as  $\Pi_I(\mathbf{Q}, \mathbf{X}) \equiv \sum_i \Pi_i(\mathbf{Q}, \mathbf{X})$ . Total industry profit  $\Pi_I(\mathbf{Q}, \mathbf{X})$  is bounded above by the monopoly profit  $\Pi^M$ , so that  $\Pi_I(\mathbf{Q}, \mathbf{X}) \leq \Pi^M$ . It follows from Dedekind completeness that every non-empty set of real numbers having an upper bound must also have a least upper bound. Thus we let  $\Pi^{sup}(\mathbf{X})$  denote the least-upper bound (or supremum) of equilibrium industry profits in subgame  $\mathbf{X}$ . Hence,  $\forall \mathbf{Q} \in SFE(\mathbf{X}), \Pi(\mathbf{Q}, \mathbf{X}) \leq \Pi^{sup}(\mathbf{X})$ . It immediately follows that for every positive number  $\forall \epsilon > 0$  there exist an equilibrium  $\mathbf{Q}_\epsilon \in SFE(\mathbf{X})$  such that  $\Pi_I(\mathbf{Q}_\epsilon, \mathbf{X}) \geq \Pi^{sup}(\mathbf{X}) - \epsilon$ . Indeed, if such an equilibrium would not exist, then  $\Pi^{sup}(\mathbf{X})$  would not be the least-upper bound of equilibrium industry profits in subgame  $\mathbf{X}$ . We now prove by contradiction that the SFE  $\mathbf{Q}_\epsilon$  is  $\epsilon$ -Pareto Optimal. Suppose it would not, then there exist an alternative SFE  $\tilde{\mathbf{Q}} \in SFE(\mathbf{X})$  such that some firm  $i$  improves its profit at least with  $\epsilon$ ,  $\Pi_i(\tilde{\mathbf{Q}}, \mathbf{X}) - \Pi_i(\mathbf{Q}_\epsilon, \mathbf{X}) > \epsilon$  while other firms  $j \neq i$  receive at least as much as before  $\Pi_j(\tilde{\mathbf{Q}}, \mathbf{X}) - \Pi_j(\mathbf{Q}_\epsilon, \mathbf{X}) \geq 0$ . This implies however that  $\Pi_I(\tilde{\mathbf{Q}}, \mathbf{X}) > \Pi_I(\mathbf{Q}_\epsilon, \mathbf{X}) + \epsilon \geq \Pi^{sup}(\mathbf{X})$ , which is impossible given the definition of  $\Pi^{sup}(\mathbf{X})$ .

**Step 3:** It follows from step 2 that we can always find an  $\epsilon$ -Pareto optimal supply function equilibrium  $\tilde{\mathbf{Q}}_\epsilon(\mathbf{X}) \in SFE_{\epsilon-PO}(\mathbf{X})$  in each subgame, while the equilibrium  $\mathbf{Q}^*$  is played along the equilibrium path  $\mathbf{X}^*$ , i.e.  $\tilde{\mathbf{Q}}_\epsilon(\mathbf{X}^*) = \mathbf{Q}^*$ . In order to prove that  $\{\mathbf{X}^*, \tilde{\mathbf{Q}}_\epsilon(\mathbf{X})\}$  is an  $\epsilon$ -PO-SPNE, it remains to show that deviations from  $\mathbf{X}^*$  in the first stage are not profitable. As the first order conditions are necessary conditions for an equilibrium, we must have that  $\tilde{\mathbf{Q}}_\epsilon(\mathbf{X}) \in FOC(\mathbf{X})$ . Definition 8 implies that firm  $i$  has no profitable deviation  $X_i(p) \neq X_i^*(p)$ , such that  $\Pi_i(\mathbf{Q}^*, \mathbf{X}^*) < \Pi_i(\tilde{\mathbf{Q}}_\epsilon(\{X_i, \mathbf{X}_{-i}^*\}), \{X_i, \mathbf{X}_{-i}^*\})$ . Thus we can conclude from Definitions 2 and 6 that  $\{\mathbf{X}^*, \tilde{\mathbf{Q}}_\epsilon(\mathbf{X})\}$  is an  $\epsilon$ -PO-SPNE. ■

## 4.4 Strategic contracting

In this subsection we solve for equilibrium contracts. In order to simplify our notation we let

$$H(x) := N(N-1) \frac{1-F(x)}{f(x)} \quad (17)$$

be the inverse hazard rate of the probability distribution of the demand shock multiplied with  $N(N-1)$ , which is a measure of the interaction effect between firms. For simplicity we set cost equal to zero in the remainder of the paper. We also make a fairly non-restrictive assumption on  $H(x)$ .

### Assumption 1

1. Production costs are zero, i.e.  $C_i(Q_i) \equiv 0, \forall i$
2. The inverse hazard rate is decreasing or mildly increasing,  $H'(\varepsilon) < 1 \forall \varepsilon \in (0, \bar{\varepsilon})$ , i.e. the hazard rate is increasing or mildly decreasing.<sup>21</sup>

Firm  $k$  maximizes profit by trading derivatives in stage 1, taking into account that the spot market outcome should satisfy the generalized Klemperer and Meyer FOC conditions (9), and that clearing of the spot market requires that spot demand must equal spot supply. Under Assumption 1 firm  $k$ 's first stage profit (16) simplifies to  $pQ_k(p)$ . Hence, firm  $k$ 's optimal contracting level is determined by the following MPEC problem.

$$\begin{aligned} \text{MPEC}(k): \quad & \max \int_0^{\bar{p}} p \cdot Q_k(p) \cdot f(\varepsilon(p)) \cdot \underbrace{u(p)}_{d\varepsilon} dp. \\ & \text{s.t.} \quad \begin{cases} \forall i \in \{1, \dots, N\} : & \sum_{j \neq i} Q'_j(p) = D'(p) + \frac{Q_i(p) - X_i(p)}{p} \\ & \varepsilon' = u(p) \\ & D(p) + \varepsilon(p) = \sum_i Q_i(p) \end{cases} \end{aligned} \quad (18)$$

Note that in the constraints of the optimal control problem we have replaced net-supply  $S_i(p)$  in the Klemperer and Meyer equation by  $Q_i(p) - X_i(p)$ .

In order to find the first stage equilibrium contracting levels, we solve for an EPEC outcome, i.e. an outcome which is optimal for each firm's MPEC problem as defined in (18). The next Proposition provides necessary first order conditions and a sufficient condition for a symmetric solution to be optimal, i.e. to be an EPEC outcome.

**Proposition 5** *[EPEC outcome] Under Assumption 1 any symmetric solution of the EPEC defined as the set of MPEC( $k$ ) problems  $k = 1 \dots N$  in Equation (18) has to satisfy the following first order conditions:*

$$H(\varepsilon(p)) = (pD(p))' + \varepsilon(p) + (N-1)^2 p \varepsilon'(p) \quad (19)$$

$$NQ(p) = D(p) + \varepsilon(p) \quad (20)$$

$$NX(p) = (pD(p))' + \varepsilon(p) - (N-1)p \varepsilon'(p) \quad (21)$$

<sup>21</sup>Note that most probability distributions that one encounters in practice, such as the normal distribution and the uniform-distribution, have increasing hazard rates, i.e. decreasing inverse hazard rates  $H'(\varepsilon) \leq 0$ .

for  $\varepsilon(p) \in [\underline{\varepsilon}, \bar{\varepsilon}]$ . Solutions to these equations are EPEC outcomes.

**Proof. Step 1** Without loss of generality we solve for the optimal contract of firm  $k = 1$ . In the optimal control problem,  $X_1(p)$  only appears in the first constraint for  $i = 1$ . This implies that this constraint is never going to be binding, as firm 1 can freely choose  $X_1(p)$  to satisfy this equation without influencing other constraints or the objective function. In the objective function and in the second and third constraints, competitors' total output matters, but not how it is divided between these firms. We can therefore sum up the remaining  $(N - 1)$  equations (for cases  $i \neq 1$ ) of the first constraint into one single constraint. Using that  $F(\bar{\varepsilon}) = 1$ ,  $\varepsilon(\bar{p}) = \bar{\varepsilon}$  and integration by parts we can now rewrite the dynamic optimization problem as follows:

$$\max \int_0^{\bar{p}} (pQ_1)' [1 - F(\varepsilon)] dp. \quad (22)$$

$$s.t. \quad (N - 1)pQ_1' + (N - 2)pQ_{-1}' = (N - 1)pD' + Q_{-1} - X_{-1} \quad (23)$$

$$Q_{-1} + Q_1 = D + \varepsilon, \quad (24)$$

where, as before, the subscript  $_{-i}$  refers to the sum of a variable over all firms, excluding firm  $i$ . Thus firm 1's expected profit is given by the integral of its marginal profit  $\frac{\partial}{\partial p} (pQ_1(p))$  at price  $p$ , weighted by  $1 - F(\varepsilon(p))$ , the probability that the realized price is larger than  $p$ , and this also makes sense intuitively.

**Step 2** We simplify the dynamic optimization problem by rewriting the constraints and then substituting them into the objective function. By adding the constraint (23) and  $N - 1$  times constraint (24) we get:

$$(N - 1)(pQ_1)' = (N - 1)(pD)' - X_{-1} + (N - 1)\varepsilon - (N - 2)(pQ_{-1})'. \quad (25)$$

We use the identity in (24) to write  $(pQ_{-1})'$  as a function of  $(pQ_1)'$ .

$$(pQ_{-1})' = (pD)' + (p\varepsilon)' - (pQ_1)',$$

which we can substitute into (25), to give an expression for the marginal profit  $(pQ_1)'$

$$(pQ_1)' = (pD)' - X_{-1} + \varepsilon - (N - 2) \cdot p \cdot \varepsilon'.$$

Substituting this marginal profit into the objective function in (22) gives the following optimization problem:

$$\max \int_0^{\bar{p}} \underbrace{\{(pD)' - X_{-1} + \varepsilon - (N - 2) \cdot p \cdot \varepsilon'\}}_{h_1(p)} [1 - F(\varepsilon)] dp. \quad (26)$$

**Step 3** We now derive the first order conditions of the optimization program (26). First we write it as the sum of two integrals:

$$\max \int_0^{\bar{p}} \{h_1(p) + \varepsilon\} [1 - F(\varepsilon)] dp - (N - 2) \int_0^{\bar{p}} p \cdot (G(\varepsilon) - G(\bar{\varepsilon}))' dp,$$

where  $G(\varepsilon) = \int_0^\varepsilon (1 - F(t))dt$ . Note that  $(G(\bar{\varepsilon}))'$  is zero and that  $\varepsilon(\bar{p}) = \bar{\varepsilon}$ . Thus the second term can be rewritten using integration by parts:

$$\max \int_0^{\bar{p}} \underbrace{\{[h_1(p) + \varepsilon] [1 - F(\varepsilon)] + (N - 2) (G(\varepsilon) - G(\bar{\varepsilon}))\}}_{\theta(p, \varepsilon)} dp.$$

The integrand only depends on  $\varepsilon(p)$  and  $p$ , so we can maximize the integral by maximizing  $\theta(p, \varepsilon)$  for each  $p$ .

$$\begin{aligned} \frac{\partial \theta(p, \varepsilon)}{\partial \varepsilon} &= (N - 1) (1 - F(\varepsilon(p))) - (h_1(p) + \varepsilon(p)) f(\varepsilon(p)) \\ &= f(\varepsilon(p)) \cdot \left[ \frac{(N - 1) (1 - F(\varepsilon(p)))}{f(\varepsilon(p))} - (h_1(p) + \varepsilon(p)) \right]. \end{aligned} \quad (27)$$

Thus the first order condition of this optimization problem is:

$$\frac{[1 - F(\varepsilon)] (N - 1)}{f(\varepsilon)} - [h_1(p) + \varepsilon] = 0. \quad (28)$$

**Step 4** We want to know under what circumstances solutions to this condition globally maximize profits. Let  $\tilde{\varepsilon}(p)$  be a solution to this equation for a given contracting choice of the competitors,  $X_{-1}(p)$ . We see from (27) that  $\frac{\partial \theta(p, \varepsilon)}{\partial \varepsilon}$  has the same sign as  $\frac{(N-1)(1-F(\varepsilon(p)))}{f(\varepsilon(p))} - (h_1(p) + \varepsilon(p))$ . It follows from Assumption 1.2 that  $H'(\varepsilon) < 1$ , so  $\frac{d}{d\varepsilon} \left( \frac{1-F(\varepsilon)}{f(\varepsilon)} \right) < \frac{1}{N(N-1)}$ . Thus we realize that for all price levels  $p$ :

$$\begin{aligned} \frac{(1 - F(\varepsilon)) (N - 1)}{f(\varepsilon)} - (h_1(p) + \varepsilon) &\leq 0 \text{ if } \varepsilon > \tilde{\varepsilon}(p) \\ &= 0 \text{ if } \varepsilon = \tilde{\varepsilon}(p) \\ &\geq 0 \text{ if } \varepsilon < \tilde{\varepsilon}(p). \end{aligned}$$

Accordingly,  $\tilde{\varepsilon}(p)$  globally maximizes  $\theta$  at each price. We can repeat the argument for any firm and thus solutions to (28) are EPEC outcomes according to Definition 8.

**Step 5.** Next we rewrite the first order condition in (28), which is a function of  $\varepsilon(p)$ , to derive a relation that can be used to determine the optimal output that firms commit to in equilibrium. Substituting the definition of  $h_1(p)$  in (26) into (28) we find:

$$\frac{[1 - F(\varepsilon)] (N - 1)}{f(\varepsilon)} - [(pD)' - X_{-1} + \varepsilon] = 0. \quad (29)$$

In a symmetric equilibrium it follows from the constraints in (24) that:

$$\varepsilon = Q_{-1}(p) + Q_1(p) - D(p) = NQ - D, \quad (30)$$

which yields (20) after some rearrangements. From (25) and 30) we have

$$\begin{aligned} NX &= NX_i = N(p \cdot D)' + N\varepsilon - N(N - 1)(pQ)' \\ &= N(p \cdot D)' + N\varepsilon - (N - 1)(p(\varepsilon + D))', \end{aligned}$$

which gives (21). (19) now follows by substituting (21) into (29). ■

The following results prove that the EPEC outcome in Proposition 5 is an EPEC equilibrium that is on the equilibrium path of an  $\epsilon$ -PO SPNE.



**Proposition 6** *The symmetric EPEC outcome in Proposition 5 is an EPEC equilibrium.*

**Proof.** First we want to verify that  $\varepsilon'(0) > 0$ . We rewrite the first order condition (19) and evaluate it at  $p = 0$ :

$$(N-1)^2 \varepsilon'(0) = \lim_{p \rightarrow 0} \frac{H(\varepsilon(p)) - \varepsilon(p) - D(p)}{p} - D'(0)$$

We use l'Hopital's rule and collect the terms with  $\varepsilon'(0)$  to find

$$\varepsilon'(0) (1 - H'^2) = -2D'(0).$$

The second factor is positive given Assumption 1.2. It follows from the assumed properties of the demand function in Section 3 that the right hand side is also positive, as  $D'(0) < 0$ . Thus we must have that  $\varepsilon'(p) > 0$ .

In the next step we show that whenever  $\varepsilon'(p) = 0$  for a given strictly positive price  $p > 0$ , it must be that  $\varepsilon''(p) > 0$ . Differentiating the first order condition (19) with respect to  $p$ , we find

$$H'(\varepsilon)\varepsilon' = (N-1)^2(\varepsilon' + p\varepsilon'') + \varepsilon' + (pD)''$$

For price levels where  $\varepsilon' = 0$ , this expression simplifies to:

$$(N-1)^2 p \varepsilon'' = -(pD)''$$

The right hand side is positive as the revenue function  $pD$  is concave according to Section 3. Hence, as prices are strictly positive, it must be that  $\varepsilon''(p) > 0$  when  $\varepsilon' = 0$ .

In the last step we show that  $\varepsilon'(p) > 0$  for all prices. Our proof is by contradiction. Assume that the inequality  $\varepsilon'(p) > 0$  is violated for some  $p > 0$ . Let  $p_0$  be the lowest price above 0 where  $\varepsilon'(p) = 0$ . Thus our assumptions would imply that  $\varepsilon'(p) > 0$  for  $p \in [0, p_0)$  and that  $\varepsilon'(p_0) = 0$ , which requires that  $\varepsilon''(p_0) \leq 0$ . However, this is impossible as we have just shown that whenever  $\varepsilon'(p_0) = 0$  it must be that  $\varepsilon''(p_0) > 0$ . Hence,  $\varepsilon'(p) > 0$  for  $p \in [0, \bar{p}]$ .

Marginal costs are zero according to Assumption 1.1, so it now follows from Definition 1 and Proposition 3 that the symmetric solution  $Q(p)$  constitutes a Nash equilibrium in the second stage. Hence, we can conclude from Proposition 4 that the symmetric EPEC outcome  $\{X(p), Q(p)\}$  is also an EPEC equilibrium. ■

**Proposition 7** *The symmetric EPEC outcome in Proposition 5 is on the equilibrium path of an  $\epsilon$ -PO SPNE for any  $\epsilon > 0$ .*

**Proof.** We know from Proposition 4 that the EPEC equilibrium is on the equilibrium path of an  $\epsilon$ -PO SPNE if (1) the monopoly payoff is bounded and (2) there exist an SFE in every subgame. We prove that both conditions hold.

Let  $D_1 = -D'(0) > 0$ . Thus it follows from the assumed properties of the demand function in Section 3 that

$$D(p) \leq -D_1 p.$$

We realize that the monopoly profit for demand  $D(p)$  is bounded by the monopoly profit for demand  $-D_1 p$ . In the latter case, the monopolist would set a monopoly price  $p = \frac{\varepsilon}{2D_1}$  and receive a monopoly profit  $\pi^M(\varepsilon) = \frac{\varepsilon^2}{4D_1}$ . In expectation this monopoly profit is:

$$\Pi^M = \int_0^{\bar{\varepsilon}} \pi^M(\varepsilon) f(\varepsilon) d\varepsilon = \frac{1}{4D_1} \int_0^{\bar{\varepsilon}} \varepsilon^2 f(\varepsilon) d\varepsilon.$$

In Section 3 we make the assumption that  $\varepsilon$  has a bounded variance, so the expected monopoly profit must also be bounded.

Finally we note that Bertrand offers constitute a Nash equilibrium in every subgame irrespective of contracting. If competitors' total offers meet maximum demand at  $p = 0$ , then the profit of a firm is always zero irrespective of its offer, and it might as well choose its supply offer such that its total output meets maximum demand at  $p = 0$ . Thus the Bertrand outcome is always an equilibrium. ■

Intuition: For a better understanding of the strategic interactions in our game, we have a brief look at firm 1's residual demand function. It is equal to total demand (term I) minus output of competitors, which is the production that they sell in the contract market (Term II) and in the spot market (Term III).

$$Q_1 = \underbrace{D + \varepsilon}_I - \underbrace{X_{-1}}_{II} - \underbrace{S_{-1}}_{III}$$

By summing constraints (23) and (24) we can rewrite the residual demand function as:

$$Q_1 = \underbrace{D + \varepsilon}_I - \underbrace{X_{-1}}_{II} - \underbrace{(N-1)|D'|p}_{III.a} - \underbrace{(N-1)Q'_1 p}_{III.b} - \underbrace{(N-2)Q'_{-1} p}_{III.c} \quad (31)$$

Competitors' net-sales in the spot market are proportional to the slope of their residual demand function, which explains terms (III.a-c). If demand is more elastic ( $|D'|$  larger in term III.a) then the output of its  $N-1$  competitors will be larger, and the residual demand that firm 1 faces decreases. Similarly, if firm 1's output is flatter ( $Q'_1$  is large in Term III.b), the output of its competitors' increases, and its residual demand decreases. Term (III.c) is an interaction effect between competitors of firm 1. If one competitor sets a flatter supply function, then the other  $(N-2)$  competitors will be more competitive, and the residual demand that firm 1 faces decreases.<sup>22</sup>

In the Allaz and Vila (1993) model firms' production does not depend on prices,  $\forall i, Q'_i = 0$ , and firm 1's residual demand function consist only of the terms I, II, and III.a. Term III.a corresponds to the Stackelberg effect of firm 1: By being

<sup>22</sup>Note that  $Q'_{-1} = \sum_{j \neq 1} Q'_j$  represents the aggregate supply slope of all  $N-1$  competitors of firm  $i$ .

a first mover in stage 1, firm 1 can affect its competitors production level in the second stage, as they will react to firm 1's output level.<sup>23</sup> Term II corresponds to the first mover effect of firm 1's competitors. As its competitors sell forward, firm 1 faces a smaller residual demand function.

The two additional terms III.b and III.c in our model are a consequence of allowing output to depend on prices. We can now obtain some intuition on the incentives of firm 1 to make output inelastic or even downward sloping. This can be seen most easily in a duopoly setting ( $N = 2$ ) in which case term (III.c) is zero. It follows from (31) that it is 'costly', either in terms of a reduced quantity or a reduced price, to set a positive slope  $Q'_1 > 0$ , because it makes its competitor's residual demand curve more elastic, which increases its output (term III.b becomes larger). Thus we would expect that firm 1 would find it optimal to keep this slope relatively small or even negative. To achieve this and still sell a significant amount, it will be optimal to produce a relatively large quantity at  $p = 0$  and then to keep output fairly inelastic or even backward bending in the whole price range. This is the result that we get in the next section.

## 5 Closed-form solutions

Relying on the propositions of section 4, this section derives closed-form solutions of our model, and discusses the welfare effect of derivatives trading.

### 5.1 Market equilibrium

In order to explicitly solve and evaluate the necessary and sufficient conditions of an EPEC equilibrium that is on the equilibrium path of an  $\epsilon$ -PO SPNE for any  $\epsilon > 0$ , we make the following simplifying assumptions:

**Assumption 2** *Production costs are zero, the demand function  $D(p) = -D_1 p$  is linear with  $D_1 > 0$  and demand shocks are Pareto distributed of the second-order, so that  $f(\varepsilon) = \beta^{1/\alpha} (\alpha\varepsilon + \beta)^{-1/\alpha-1}$  for  $\varepsilon > 0$ , where  $\beta > 0$  and  $\alpha \in (-\infty, \frac{1}{N(N-1)})$ .*

The Pareto distribution of the second-order is a family of probability distributions with a wide range of properties (Johnson et al., 1994). For example, for  $\alpha = 0$  it gives the exponential distribution and for  $\alpha = -1$ , the uniform distribution.

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<sup>23</sup>This term can be understood better by looking at a standard Stackelberg game with one leader and  $N - 1$  followers. The leader sets output, taken into account the subsequent reaction of its followers. Each follower sets its output such that its marginal revenue equals marginal costs, which is zero in our model. Hence, for each follower  $j$ ,  $Q_j + pD' = 0$ . The output of one follower is  $Q_j = |D'|p$ , and total output of all followers is given by  $(N - 1)|D'|p$ .

**Proposition 8** [Closed-Form] Under Assumption 2 the unique symmetric EPEC equilibrium defined by the conditions in Proposition 5 is given by:

$$\varepsilon(p) = N \frac{\beta(N-1)}{1 - \alpha(N-1)N} + \frac{2}{1 - \alpha(N-1)N + (N-1)^2} D_1 p \quad (32)$$

$$Q(p) = \frac{\beta(N-1)}{1 - \alpha(N-1)N} + \frac{\alpha(N-1) - (N-2)}{1 - \alpha(N-1)N + (N-1)^2} D_1 p \quad (33)$$

$$X(p) = \frac{\beta(N-1)}{1 - \alpha(N-1)N} + \frac{2\alpha(N-1) - 2N + 2}{1 - \alpha(N-1)N + (N-1)^2} D_1 p. \quad (34)$$

The equilibrium is on the equilibrium path of an  $\epsilon$ -PO SPNE for any  $\epsilon > 0$ .

**Proof.** Under Assumption 2 we have  $\frac{1-F(\epsilon)}{f(\epsilon)} = \alpha\epsilon + \beta$  (Holmberg, 2009) and  $D(p) = -D_1 p$ . Thus (19) can be simplified as follows:

$$N(N-1)\beta + 2D_1 p = (N-1)^2 p \epsilon' + \epsilon [1 - N(N-1)\alpha].$$

Thus

$$a\epsilon + p\epsilon' = g(p) \quad (35)$$

with

$$a = \frac{1 - \alpha(N-1)N}{(N-1)^2} \quad (36)$$

and

$$g(p) = \frac{N}{N-1}\beta + \frac{2}{(N-1)^2} D_1 p. \quad (37)$$

We multiply both sides of (35) with the integrating factor  $p^{a-1}$  and then integrate both sides. As long as  $a > 0$  or equivalently  $\alpha < \frac{1}{(N-1)N}$ , we have that  $p^a \epsilon(p)$  is zero at  $p = 0$ , so

$$\varepsilon(p) = p^{-a} \int_0^p g(t) t^{a-1} dt.$$

Substituting  $a$  and  $g(t)$  by their definitions in (36) and (37) and solving for this integral gives us equation (32). The optimal supply and contracting functions in (33) and (34) then follows from substituting (32) into (20) and (21), respectively.

It follows from Johnson et al. (1994) that the Pareto distribution of the second-order has a finite variance when  $\alpha < \frac{1}{N(N-1)}$ . We also note that under Assumption 2 we have  $H'(\epsilon) = N(N-1)\alpha < 1$ . Thus it follows from Propositions 5-7 that the linear solution is an EPEC equilibrium that is on the equilibrium path of an  $\epsilon$ -PO SPNE for any  $\epsilon > 0$  ■

Figure 4 illustrates the results of Proposition 8 for the special case where  $N = 2$  and the demand shock is uniformly distributed.

□ We notice that with linear demand, the contracting and output functions are linear for a Pareto distribution of the second order. The net-supply  $S(p) = Q(p) - X(p)$  is up-ward sloping, but the contracting function is downward sloping; producers sell forward contracts and buy call options for strike prices above zero. The output function is also backward bending for  $N > 2$  or when  $\alpha < 0$ .

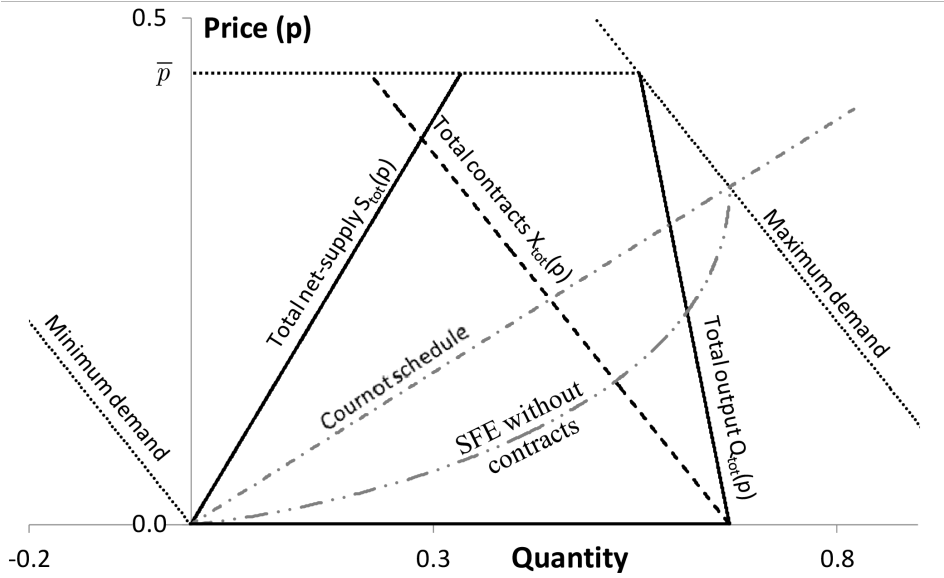


Figure 4: Total contracting and output for the SPNE of a duopoly market with uniformly distributed demand.

Hence firms produce less, although the demand shock increases. As a result prices increase steeply as demand shocks increase. Even in the alternative case where total output is forward bending (duopoly  $N = 2$  and  $\alpha \geq 0$ ), the total output curve is still very steep. The slope of the total output as a function of price is less than  $|D'|$ , the slope of monopoly output[.]

As demand shocks become more uncertain ( $\alpha$  increases), the anti-competitive consequences of contracts are mitigated ( $Q'(p)$  becomes less negative).<sup>24</sup> However, even for the highest level of demand uncertainty that we consider  $\alpha = \frac{1}{(N-1)N}$  total supply remains backward bending for  $\forall N > 2$ . In the limit where demand is certain ( $\alpha \rightarrow -\infty$ ), total supply will converge to  $NQ(p) \rightarrow -|D'|p$ , which is less than the monopoly output. Thus for sufficiently small  $\alpha$ , social welfare is lower than in a monopoly market without contracts.

It is also straightforward to verify that total forward sales ( $X(0)$ ) increases with the number of firms. This ensures that the market becomes more competitive for low shock outcomes. However, the total output function will bend backwards more, when the number of firms increase;  $NQ'(p)$  decreases with more firms in the market. Thus for  $\alpha \geq 0$  (when the support of the shock density is infinite) the market will be less competitive for the highest shock outcomes if there are more firms in the market. Hence, our finding that firms have incentives to use option contracts to commit to fairly inelastic or even a backward bending output, becomes more apparent in markets with more firms. We attribute this to the interaction effect between competitors, term III.c in (31).

<sup>24</sup>For  $\alpha \leq 0$ , the demand shock range is  $\left[0, \frac{\beta}{|\alpha|}\right]$ , so a less negative  $\alpha$  increases the range of demand shocks. For  $\alpha \geq 0$ , a larger  $\alpha$  increases the thickness of the tail of the demand density (Holmberg, 2009).

## 5.2 Welfare effects

**Proposition 9** [Deadweight Loss] *The expected deadweight loss  $L$  for the equilibrium in Proposition 8 is:*

$$L = \frac{\beta^2 (1 - \alpha (N - 1) N + (N - 1)^2)^2}{4D_1 (1 - \alpha) (1 - 2\alpha) (1 - \alpha (N - 1) N)^{2-1/\alpha}}. \quad (38)$$

**Proof.** The market clears when  $NQ(p) = \varepsilon - D_1 p$ , so using (33) it follows that:

$$P(\varepsilon) = \begin{cases} A(\varepsilon - \varepsilon_0) & \text{if } \varepsilon > \varepsilon_0 \\ 0 & \text{if } \varepsilon \leq \varepsilon_0 \end{cases}$$

with  $\varepsilon_0 \equiv \frac{-\beta}{\alpha - \frac{1}{N(N-1)}}$  and  $A = \frac{1 - \alpha(N-1)N + (N-1)^2}{2D_1}$ . As in Holmberg and Newbery (2010), the welfare loss for a given demand shock  $\varepsilon$  is the deadweight-loss:

$$L(\varepsilon) = \frac{P^2(\varepsilon) D_1}{2} = \begin{cases} A^2 \frac{D_1}{2} (\varepsilon - \varepsilon_0)^2 & \text{if } \varepsilon > \varepsilon_0 \\ 0 & \text{otherwise.} \end{cases}$$

while the expected welfare loss is given by

$$L = E_\varepsilon [L(\varepsilon)] = \int_{\varepsilon_0}^{\bar{\varepsilon}} L(\varepsilon) f(\varepsilon) d\varepsilon.$$

Define  $H(x)$  as the third-order integral of  $f(x)$ , so that  $H(x) = \frac{-\beta^2}{(\alpha-1)(2\alpha-1)} \left(\frac{\alpha x + \beta}{\beta}\right)^{2-\frac{1}{\alpha}}$  and  $H'''(x) = f(x)$ . We can now evaluate the expected loss by twice integrating by parts.

$$\begin{aligned} L &= \underbrace{[L(\varepsilon)H''(\varepsilon)]_{\varepsilon_0}^{\bar{\varepsilon}} - [L'(\varepsilon)H'(\varepsilon)]_{\varepsilon_0}^{\bar{\varepsilon}}}_{=0} + L''(\varepsilon) [H(\varepsilon)]_{\varepsilon_0}^{\bar{\varepsilon}} \\ &= -A^2 D_1 H(\varepsilon_0) = A^2 D_1 \beta^2 \frac{(1 - \alpha N(N - 1))^{1/\alpha-2}}{(1 - \alpha)(1 - 2\alpha)} \end{aligned} \quad (39)$$

Note that the first two terms are zero as long as  $\alpha < \frac{1}{2}$ , because  $\bar{\varepsilon} = \frac{-\beta}{\alpha}$  if  $\alpha < 0$  and  $\bar{\varepsilon} = \infty$  otherwise. Welfare losses are quadratic in  $\varepsilon$ , therefore  $L''^2 D_1$  is a constant. We get (38) by substituting  $A = \frac{1 - \alpha(N-1)N + (N-1)^2}{2D_1}$  into (39). ■

It follows from (38) that the market becomes perfectly competitive (no welfare losses) in the limit when  $N \rightarrow \infty$  and  $\alpha \leq 0$ .<sup>25</sup>

**Proposition 10** *Under Assumption 2, the expected deadweight loss for the equilibrium in Proposition 8 decreases with the number of symmetric firms.*

**Proof.** The welfare loss is given by (38). Lemma 2 in the Appendix shows that  $\frac{1 - \alpha(N-1)N + (N-1)^2}{(1 - \alpha(N-1)N)^{1-(1/2\alpha)}}$  is decreasing with respect to  $N$  which proves the result. ■

<sup>25</sup>Note that Assumption 2 does not allow for  $\alpha > 0$  when  $N \rightarrow \infty$ .

**Proposition 11** [*Welfare Comparison*] *Expected welfare is lower in the unique symmetric EPEC equilibrium than in a standard Cournot model without contracting where demand shocks are realized before firms choose production, provided that  $N = 2$  and Assumption 2 is satisfied.*

**Proof.** From the first-order condition of the Cournot market with certain demand it follows that the total duopoly output is:

$$Q_{tot} = 2D_1p.$$

The market clears when  $Q_{tot} = \varepsilon - D_1p$ , so  $P_{Cournot}(\varepsilon) = \frac{\varepsilon}{3D_1}$ . As in Holmberg and Newbery (2010), the deadweight loss for a given  $\varepsilon$  is:

$$L_{Cournot}(\varepsilon) = \frac{P_{Cournot}^2(\varepsilon) D_1}{2} = \frac{\varepsilon^2}{18D_1}.$$

We can calculate the expected welfare losses  $L_{Cournot} = E_\varepsilon [L_{Cournot}(\varepsilon)]$  as in Proposition 9 by twice integrating by parts:

$$L_{Cournot} = L''_{Cournot}(\varepsilon) [H(\varepsilon)]_0^\varepsilon = \frac{1}{9D_1} H(0) = \frac{1}{9D_1} \frac{\beta^2}{(1-\alpha)(1-2\alpha)}.$$

Comparing the welfare losses in the Cournot game above and in the Supply function model in Proposition 8 we find that:

$$\frac{L}{L_{Cournot}} = 9 \frac{(1-\alpha)^2}{(1-2\alpha)^{2-1/\alpha}},$$

which is larger than 1 according to Lemma 1 in the Appendix. ■

Prices for a symmetric SFE without contracts are below the Cournot schedule (Klemperer and Meyer, 1989; Green and Newbery, 1992), i.e. prices in a Cournot equilibrium where demand shocks are realized before firms set production quantity. Thus Proposition 11 has the following implication:

**Corollary 1** *The introduction of a derivatives market will lower welfare, provided that  $N = 2$ , Assumption 2 is satisfied, and firms play the unique symmetric EPEC equilibrium after the reform.*

This result contrasts with Holmberg's (2011) two-stage model with forward contracting and supply function competition, where the introduction of a forward market weakly improved competition for cases when marginal costs are constant up to a capacity constraint.

## 6 Conclusion

Commodity derivatives such as forwards and call options are very useful hedging instruments. However, in an oligopoly market they will also be used strategically. Seminal results by Allaz and Vila (1993) indicate that their strategic effect is

pro-competitive. However, Mahenc and Salanié (2004), Ferreira (2003) and our new study show that strategic contracting might as well have anti-competitive consequences.

Solving for an equilibrium of a two-stage game with derivative trade followed by spot market competition, we show that risk-neutral producers sell forward contracts and buy call option contracts. This trading strategy is called a bear call spread. Traders use it when speculating on a lower commodity price, but it also commits strategic firms to a fairly inelastic or even downward sloping supply function in the spot market. This makes competitors' residual demand less elastic, so their mark-ups are high and the strategic firm can increase its profit at competitors' and consumers' expense. The forward sales improve competition for low demand realizations, but the option contracts reduce competition for high demand realizations. Hence commodity derivatives are pro-competitive for low demand, but anti-competitive during high demand. In a duopoly market, the second effect outweighs the first and total surplus decreases when the financial market is introduced.

We show that the anti-competitive effects worsen when demand uncertainty is smaller. When demand uncertainty is sufficiently small, strategic contracting results in spot prices above monopoly prices, hurting both producers and consumers. When demand uncertainty becomes larger, it is optimal for firms to offer supply functions that have a less negative slope, as this allows them to benefit more from both high and low demand realizations. Thus to avoid the anti-competitive effect of speculation, this suggests that option contracts should not be traded near delivery because firms then have a good estimate of demand. Alternatively, the same option contract or supply function should be valid for several delivery periods in order to increase demand variation.<sup>26</sup>

Total forward sales increase in a less concentrated market (more firms), which improves the low demand pro-competitive effect. In expectation more firms in the market reduce welfare losses, even if the anti-competitive effect at high demand becomes more pronounced because of strategic interaction effects. If a negative supply slope induces a competitor to choose a less competitive supply function with a less positive slope, then this response will induce other competitors to further increase their mark-ups. Hence, the incentives for setting an inelastic or even downward sloping supply function is greater in less-concentrated markets. Unlike spot markets with Cournot competition (Willems, 2005), our results do not depend on whether option contracts are financial or physical.

Oren (2005) recommends that electricity markets should use call options with high strike prices in order to steady the revenue flow of peak power plants in electricity markets. These plants have a high marginal cost and are used infrequently, so such a recommendation makes much sense from a hedging perspective. But our results indicate that there are also drawbacks with introducing option contracts with high strike prices, because especially large producers with market power have

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<sup>26</sup>This is for example the case in the PJM market, where producers' offers are fixed during the whole day to meet a wide range of demand outcomes. PJM is the largest deregulated wholesale electricity market, covering all or parts of 13 U.S. states and the District of Columbia.



incentives to misuse them, i.e. to buy call options at high strike prices. To avoid these pitfalls, market regulators should take measures to increase demand variation for option contracts and market monitors should carefully scrutinize how such contracts are used. Especially, situations where producers use contracts in a speculative manner should be examined.

In our model producers are risk neutral and arbitrage in the financial market is perfect. Therefore, commitment with financial derivatives is costless. As this is not the case in practice, our results should be seen as a limiting case. With risk aversion, firms are expected to reduce tail risk and to hold contracting portfolios that are closer to their actual output, and therefore to offer supply functions that are less steep. Thus contracting should be pro-competitive with sufficient risk-aversion. Also transaction costs in the financial market are likely to reduce the profitability of speculative positions. Considering such imperfections in contracting is likely to reduce the anti-competitive effect for high demand and the pro-competitive effect for low demand realizations.

In practice firms could also use other commitment tactics than financial contracts, for instance by delegating decisions to managers, merging with downstream firms, and making irreversible investments. We believe that the main intuition of our paper, that firms would like to commit to downward sloping supply functions, remains valid in those settings. In this sense our result has parallels in Zöttl (2010), who models the strategic (irreversible) investments of firms, where firms compete in quantities in a spot market with random demand. He shows that firms will over-invest in technology with low marginal costs (base-load), but choose total investment capacities that are too low from a welfare viewpoint.

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## Appendix

**Lemma 1** For  $\alpha \in (-\infty, \frac{1}{2})$  it must be that  $9 \frac{(1-\alpha)^2}{(1-2\alpha)^{2-1/\alpha}} > 1$

**Proof.** The left hand side of the inequality can be written as  $9 \exp(z(\alpha))$  with

$$z(\alpha) = [2 \ln(1 - \alpha) + (\frac{1}{\alpha} - 2) \ln(1 - 2\alpha)].$$

We now prove that  $\alpha = 0$  is a minimum of the function  $z(\alpha)$ . By differentiation of  $z$  with respect to  $\alpha$  we obtain that:

$$\alpha^2(1 - \alpha) \frac{dz}{d\alpha} = -2\alpha - (1 - \alpha) \ln(1 - 2\alpha)$$

Love (1980) shows that  $\frac{x}{1+\frac{1}{2}x} < \ln(1+x)$  if  $x > 0$  and  $\frac{x}{1+\frac{1}{2}x} > \ln(1+x)$  if  $-1 < x < 0$ . From this it directly follows that  $\frac{dz}{d\alpha} > 0$  if  $.5 > \alpha > 0$  and  $\frac{dz}{d\alpha} < 0$  if  $\alpha < 0$ . The minimum of  $z$  is therefore achieved at  $\alpha = 0$ . Using l'Hopitals' rule it can be shown that  $z(0) = -2$ . Thus the minimum of  $9e^{z(\alpha)}$  is  $9e^{-2}$ , which is larger than 1. ■

**Lemma 2** For  $\alpha \in (-\infty, \frac{1}{2})$ , and under Assumption 2, it must be that  $\xi(N, \alpha) = \frac{1-\alpha(N-1)N+(N-1)^2}{(1-\alpha(N-1)N)^{1-(1/2\alpha)}}$  is decreasing with respect to  $N$ .

**Proof.** The partial derivative of this expression with respect to  $N$  is

$$\frac{\partial \xi(N, \alpha)}{\partial N} = - \frac{\varpi(N, \alpha)}{2(1 - \alpha(N-1)N)^{\frac{4\alpha-1}{2\alpha}}}$$

with  $\varpi(N, \alpha) = (2N - 5N^2 + 2N^3 + 2) - \alpha(N-1)(2N^2 - 3N + 2)$ . As the denominator is always positive, we need to show that  $\varpi$  is positive as well. We have

$$\frac{\partial \varpi(N, \alpha)}{\partial \alpha} = -(N-1)(2N^2 - 3N + 2) \leq 0$$

for  $N \geq 2$ . According to Assumption 2 we have  $\alpha < \frac{1}{N(N-1)}$ , so  $\frac{1}{2}$  is an upper boundary for  $\alpha$  and  $\varpi(N, \alpha)$  is bounded from below by  $\varpi(N, \frac{1}{2})$

$$\varpi(N, \alpha) > \varpi(N, \frac{1}{2}) = \frac{1}{2}(N-2)(N+1)(2N-3) \geq 0$$

for  $N \geq 2$ , which establishes the result. ■