

# Minimum uplift assignment of transmission expansion costs to beneficiaries

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## Abstract

In coming years, electric transmission networks will likely be expanded at an unprecedented rate to accommodate growth in renewables, while a significant fraction of aging infrastructure will also be replaced. Plans have already been developed to build a large amount of new transmission in Texas. However, these plans were developed without formal electric transmission network optimization methods, and the intention is to socialize the cost of transmission expansion. In addition to the need to adopt formal optimization methods, there is a pressing need to better relate drivers of transmission expansion to charging of beneficiaries. In this paper, we use a dual of the integer optimization problem of expanding transmission in order to set prices that “cover” as much of the total expansion cost as possible using linear prices. The remaining cost, due to the duality gap, could be socialized. This approach would explicitly price a much larger fraction of the total costs than current approaches, which often default to socialization, improving incentives for location of new generation and incremental demand. Basic theory and several small examples will be developed.

*Key words:* Transmission expansion, cost allocation. JEL Classification codes: D92, H54, L52, L94.

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## 1. Introduction

Electric transmission planning and cost allocation remain problematic issues in the electricity industry [7, 49]. In the context of transmission planning, there have been many decision-support tools and formulations of “optimal” electricity transmission expansion planning. However, as pointed out in [105], in practice “electricity regulators engage in transmission expansion planning without a theory-based planning tool,” and then proceed to allocate the costs of the resulting transmission. For example, the Federal Energy Regulatory Commission (FERC) has recently promulgated Order 1000 [122], updating earlier orders on cost allocation and encouraging efficient planning [122, ¶11 and ¶586], but without an underpinning theory of systematic planning to evaluate efficiency or to support that cost allocation.

Building on existing literature, we postulate the existence and use of a transmission expansion planning methodology that seeks to maximize the benefits minus the costs of transmission expansion [7]. The transmission is planned for completion at some future date or over some future time horizon, with requirements for that transmission typically derived in part from a forecast of future “economic and demographic

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variables” [52, §2] and from future generation plans. We seek prices that provide a unified approach to allocating the cost of that transmission capacity that is consistent with the issues driving the need for the transmission expansion, whether those issues are related to “reliability requirements” for delivery of power to load, “policy upgrades” for integrating renewables, “economic upgrades” with an explicit willingness-to-pay for transmission services, or a combination of these or other needs for transmission expansion. Our goal is to avoid socializing costs where this can be avoided.

In return for paying for this capacity, market participants receive property rights. Our primary focus will be electricity systems having centralized energy markets and locational marginal prices. In this context, the natural property rights are financial transmission rights (FTRs), which are claims to congestion rent in the energy market. For example, an “obligation” FTR between “injection” and “withdrawal” buses provides a claim to the difference between the locational price at the withdrawal bus minus the locational price at the injection bus, multiplied by the quantity associated with the FTR [55, 53]. Because of “reliability” mandates, lumpiness, and economies of scale, some—perhaps most—expansions will cost more than the value in the energy market of the FTRs produced [91]. Consequently, although we will avoid *broad* socialization wherever possible, there will still be a need to compel *specific* market participants to purchase at least some of the FTRs.

We do not directly treat the incentives faced by transmission planners to plan efficient augmentation of the network nor the incentives to builders to minimize construction costs. These issues are significant, and, for example, [42, 126, 127, 54] treat incentive mechanisms; however, these incentives are beyond the scope of this paper. Instead, our focus is on developing a unified approach to allocating the cost of transmission expansion whether that expansion is mandated due to “reliability” or “policy” criteria or is on the basis of an explicit willingness-to-pay in an auction process. Implicit in this assumption is some mechanism, possibly analogous to those described in [42, 126, 127, 54], that provides the incentives for a transmission planner to engage in optimal planning using data derived from efficient transmission construction practices in order to either minimize the costs of transmission expansion or optimally trade-off the costs of transmission expansion against the benefits, and also implicit is some mechanism, such as an auction [106, §II.B], to provide incentives for efficient construction of that plan.

The issue of transmission expansion has assumed greater importance in recent years, motivating this paper. For example, in Europe, North America, and Australia, there is now significant transmission expansion being contemplated or built [122, ¶2, ¶17, ¶29], after long periods of relatively little electric transmission system expansion, and a significant amount of existing transmission infrastructure is reaching or is beyond its service life. Furthermore, the great increase in expectations for integration of renewable resources throughout the world is prompting plans for significant transmission expansion [13, 118, 82, 120]. Even in countries, such as Spain, that have so far integrated considerable wind generation resources without the need for significant transmission expansion, further expansion of renewable capacity is likely to involve transmission expansion.

Increased transmission capacity may involve a combination of development of new transmission corri-

dors together with upgrades on existing corridors, potentially using recently developed conductors or other technologies [11][35, appendix D]. Some of the upgrades will typically address voltage or transient stability issues; however, the primary focus of this paper is on building transmission to satisfy so-called  $N - 1$  thermal contingency limits under the explicit assumption, articulated in [62], that the total transmission expansion costs are primarily driven by the need to satisfy thermal contingency limits. This focus is consistent with the typical ordering of economic significance of these issues. However, as observed in [72, section 7.2][17], it is important to recognize that resolution of these other issues may require considerable further analysis and additional transmission upgrades, even if these additional transmission upgrades ultimately form only a relatively small fraction of the total upgrade costs.

The analysis to satisfy the constraints besides the thermal contingency limits and the additional costs of equipment or upgrades to satisfy these constraints will not be explicitly treated in this paper. Despite our *general* desire to avoid socialization, we argue that these additional costs should be socialized to load or load serving entities under the assumptions that:

- under North American Reliability Corporation standards, many of these constraints are articulated around deliverability of power to load during extreme, stressed operating conditions involving multiple outages, and
- the additional costs are relatively small compared to the costs of building transmission to satisfy the thermal contingency limits.

Even putting aside these additional costs, the pricing of transmission system augmentation is complicated by the lumpiness of transmission expansion. Our approach to this issue will be to follow the work of Gribik, Hogan, Madrigal, Ring, and others that seeks linear prices that minimize the energy “uplift” in operation of an electricity market [50, 58, 74, 98]. We will take an analogous approach by defining a suitable dual problem to the transmission expansion problem that results in the minimum remaining costs of transmission expansion that are not explicitly priced. As with the costs of satisfying constraints other than the thermal contingency constraints, we will argue for socialization, or “uplift,” of these remaining costs.

Our approach to transmission prices contrasts with that of, for example, [91, 108, 126, 127, 54, 104, 57, 105] that consider a two-part tariff consisting of, for example, a term reflecting congestion rent plus a “complementary charge” that recovers the rest of the capital costs. Instead, we explicitly seek to cover as much of the cost of transmission as possible with a “single-part” tariff and minimize the complementary charge. In doing so, the resulting single-part prices for new transmission may exceed the value of the FTRs created by that new transmission [60], necessitating compulsion in the payment for those rights. This will be the case for what we will define as “reliability” and “policy upgrades.” Nevertheless, we hope that there are cases where prices that reflect transmission construction costs will be comparable to the payoffs from the FTRs, allowing for voluntary “economic upgrades.”

In jurisdictions such as ERCOT, transmission expansion costs are effectively all socialized. Although

there would still be some socialization of costs under the approach described in this paper, the level of socialization would be, by design, minimized. It will turn out that the key to allocation of costs is an optimization framework for transmission expansion. Even if systematic transmission expansion approaches only yield modest cost savings compared to *ad hoc* approaches to transmission planning, systematic optimization approaches allow for systematic, consistent pricing of transmission that minimizes socialization.

Our focus will be on transmission expansion. However, in the spirit of [85], it may be possible to include both generation and transmission expansion in the same framework. Moreover, the interaction of generation and transmission expansion would be better captured if they are both represented in a single framework and we believe that such a generalization is also possible. Our default assumption is that the transmission itself is built by a regulated entity; however, “merchant” transmission may also be possible in this context [67, 54, 71, 60].

Finally, we acknowledge that one of the thorniest issues in transmission expansion is the siting of new lines, particularly lines that cross jurisdictional boundaries such as state and country borders [112, 14, 15]. We do not treat this issue. Moreover, we do not treat the interaction between planning authorities in different regions.

The organization of the rest of this paper is as follows. Section 2 provides further context and a literature survey on various aspects of transmission and generation planning, property rights, and cost allocation. Section 3 then formulates the transmission expansion and cost allocation problem. Section 4 applies the formulation to several small examples. Section 5 concludes.

## 2. Context and literature survey

In this section, we provide context for the formulation and a literature survey, discussing: expansion planning in Section 2.1, transmission construction costs in Section 2.2, a taxonomy of transmission expansion in Section 2.3, regulatory institutions in Section 2.4, property rights in Section 2.5, uplift in electricity markets in Section 2.6, and finally summarize the interaction of upgrades, property rights, and cost allocation in Section 2.7.

### 2.1. Expansion planning

#### 2.1.1. Generation

Although the main focus of this paper is on transmission expansion, we will very briefly review generation expansion in this section (and complement the discussion with a review of joint transmission and generation expansion in section 2.1.3.) Generation expansion planning has been explored in both the academic literature and also implemented in commercial tools.

A typical formulation considers the amount of generation capacity to be built for some future year, assuming a possibly uncertain forecast of electrical demand [28]. Some approaches, such as the “screening curve,” model generation capacity with a continuous variable [63, 96]. Similarly, linear programming has

been applied to generation expansion planning [114]. However, because there are both discrete decisions—including the choice to build or not build a generator—and also continuous decisions—such as the dispatch of the system—generation expansion is most naturally formulated as a mixed integer problem [28, 52]. Various models have incorporated integer variables and used approaches such as Benders decomposition [44] to decompose the problem for computational tractability [46, 47]. Because the generation sector may be imperfectly competitive in restructured electricity markets, the generation expansion literature also includes models of strategic interaction [78].

There are also a number of commercial implementations of generation expansion planning techniques that use formal optimization techniques. A review of several of these commercial models appears in [39].

### 2.1.2. Transmission

As mentioned in Section 1, various formulations for optimizing transmission expansion have appeared in the literature [68]. Three of the earliest decision-support tools are [64, 31, 90], while one of the earliest approaches to systematic transmission optimization appears in [125, 124], which formulated the transmission expansion problem as a linear and as a linear mixed integer program. Subsequently, various optimization approaches have been described in the literature with the goal of performing systematic transmission expansion, aiming to either expand the transmission system to meet requirements at least cost, or expand the transmission system to minimize the cost of expansion minus the benefits of that expansion, or achieve a related criterion assuming a typically increasing electrical demand forecast over a time horizon [52, 68].<sup>1</sup>

Similarly to generation expansion planning, because there are both discrete decisions—including the choice to build or not build a line—and continuous decisions—such as the dispatch of the system—transmission expansion is also most naturally formulated as a mixed integer problem. Furthermore, although operational decisions in electricity systems in general involve non-linearities and discrete decisions and some formulations have incorporated non-linearities [129, 109, 24, 16, 17], it is typical and natural to approximate future operational decisions in a planning context using either or both linearization and continuous relaxations [52, §4]. Linearization of the power flow equations results in the “DC power flow approximation” [128, section 4.1.4], although some transmission planning formulations simplify even further to transportation models [103]. In a model with integer transmission expansion variables and the DC power flow approximation, there is still a non-linear interaction between the continuous and discrete variables; however, this can be treated using “big  $M$ ” approach, which results in a mixed integer linear programming formulation [46, §4.8.4][47, 113, 125, 34, 1].

Consequently, mixed integer linear programming approaches have been used for transmission expansion planning, as in [125, 45, 46, 47, 110, 5, 1, 29, 24], typically with the transmission expansion modeled by

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<sup>1</sup>References [54, 104, 105] consider incentives to transmission companies to expand transmission towards optimality, given a fixed level of electrical demand. Although this is an interesting special case, our focus is on the more general situation where transmission expansion is contemplated to allow for increased demand or for increased supply of, for example, renewable resources.

discrete variables, with all other decisions modeled by continuous variables, and with the objective and all constraints approximated as linear or affine. As with generation expansion, Benders decomposition [44] is a natural approach to transmission expansion planning problems since if the “higher” level discrete expansion decisions are fixed then the remaining “lower” level linearized and continuous approximation to the operations problem is convex. If load shedding is modeled with an explicit “value of lost load” then the operations problem is always feasible, so only “optimality” cuts need be represented into the “higher” level problem [88, 46, 47, 89, 101, 102, 19, 43, 95]. Nevertheless, the computational complexity of mixed integer linear programming, even with decomposition approaches, has prompted various tailored and heuristic approaches including:

- genetic algorithms [107, 40, 59];
- simulated annealing [100, 41];
- greedy algorithms [18, 37];
- the use of sensitivities [76, 69]; and
- local search [69, 109, 80, 17].

More recently, the availability of greatly increased computing power and more capable mixed integer linear programming software has enabled larger scale problems to be solved directly, including fuller consideration of issues such as:

- uncertainty of future scenarios [28, 30, 38, 20, 95], including uncertainty in growth of wind generation capacity or exogenous issues such as carbon policy [123],
- optionality [116, 123],
- uncertainty and variability of wind production and demand [116, 87], and
- strategic interaction in transmission expansion [26, 27].

However, computational complexity remains a significant issue for practical solution of large-scale problems and there do not appear to be any commercial models available to perform large-scale transmission expansion planning [39]. The published models typically focus on building transmission to meet static requirements in an assumed future “test year,” without consideration of the dynamics of expansion over successive years [107]. Moreover, many of the various issues that must be considered in detailed transmission planning, including thermal contingency constraints on transmission, contingency constraints on generation, deliverability to load under extreme conditions, and multiple outages, are typically omitted from these formulations. Amongst these issues, thermal contingency constraints on transmission are necessary, at least, to represent operational practice under normal conditions and likely drive much of the economically significant decisions [62]. Our formulation will be capable of representing thermal contingency constraints and we will illustrate this with one of the examples in Section 4.

### *2.1.3. Transmission and Generation*

Transmission expansion interacts with generation expansion in that new transmission can enable development of locationally restricted resources, such as renewable resources [13, 118, 120]. Some planning formulations have considered joint generation and transmission expansion [89, 107, 77]. In the context of restructured electricity markets, there is typically a separation of the transmission planning role from that of planning and investment in generation, increasing the uncertainties and decreasing the coordination between generation and transmission planning [119, 99], and as discussed in Section 2.1.1, the generation sector is typically imperfectly competitive. Consequently, much of the recent work in joint generation and transmission expansion has considered the strategic interaction between imperfectly competing generation investors and the planning of the transmission system [111, 112, 78, 94, 93]. A typical formulation has three levels, with transmission planning at the “highest” level, generation planning at the “middle level,” and clearing of the energy market and operations at the “lowest level.” Various approaches include simplifying assumptions about strategic interaction at one or other of the levels, such as assuming that the energy market is competitive [94, 94], and simplifying the representation of the transmission constraints, such as ignoring thermal contingency constraints.

### *2.1.4. Planning in practice*

The literature formulating systematic approaches to transmission expansion dates to the 1980s. However, for various reasons, relatively little new transmission construction took place in industrialized countries in the 1980s and 1990s. Occasional, sporadic construction of new lines involved relatively few alternatives, and the detailed focus on the specific driver for an expansion meant that only a handful of related alternative construction plans were compared at any given time [10]. Since expansion was sporadic, most planning was on a line by line basis, although a given expansion might include multiple smaller upgrades of existing lines and associated equipment in addition to the construction of a new line.

Moreover, in the United States, the jurisdictional split between the FERC and state public utility commissions has resulted in even less new inter-state transmission than intra-state transmission [14, 15]. Particularly in the context of electricity restructuring since the 1990s, most new capital formation in the electricity industry has been in the generation sector rather than in the transmission and distribution sectors.

Demand growth, the need to replace aging infrastructure, and the desire to integrate remote renewable resources, has significantly changed this situation beginning at around the turn of the century and accelerating recently, both in the United States and internationally [57]. For example, in California, transmission is being expanded to access potential wind resources in the Tehachapi area [35, page 29]. As another example, in the Electric Reliability Council of Texas (ERCOT), a major transmission expansion has begun in order to support integration of multiple renewable energy resources in the ERCOT “Competitive Renewable Energy Zones” (CREZ). The CREZ are generally remote from demand centers so that there are many possible transmission corridor choices.



Similarly, in South America, expansion of non-hydro renewables has also increased the pace of transmission expansion recently [106]. In the case of Brazil, systematic optimization approaches have been applied to transmission expansion [106, §II.B]. Systematic approaches have also been used in Spain [69, §1].

However, it appears that relatively little new transmission has been planned in the United States using systematic optimization approaches, despite the several formulations developed over the last two decades [79, 62]. For example, the “Transmission Economic Assessment Methodology” (TEAM) developed in California includes modeling of interactions between transmission expansion and generation alternatives, including strategic behavior [22, chapter 4][4]. It has an explicit framework that considers the benefits of transmission. However, the TEAM methodology does not apparently involve systematic optimization over multiple transmission line expansion alternatives.

As another example, although several major alternative plans were compared in the CREZ transmission optimization study performed by the ERCOT Independent System Operator (ISO) with input from various parties [118, section II.b], there appears to have been little or no systematic optimization of the choice of the various detailed transmission elements in the overall plan beyond a trial and error process [118, page 12]. Nevertheless, the study produced a plan that satisfied the general requirements of deliverability of wind, involving both initial consideration of the thermal contingency constraints and a subsequent detailed modeling process that incorporated additional issues, such as voltage and other constraints [72, section 7.2].

As part of the increased pace of transmission construction, there is also a move towards consideration of multiple coordinated goals in the planning of transmission expansion. For example, the CREZ transmission study considered accessing multiple regions in West Texas. As another example, the Southwest Power Pool “balanced portfolio” approach advocates baskets of projects [115]. Given the move to consideration of multiple goals, the concomitant need to consider various possible lines, and the value of also considering multiple smaller upgrades, current planning practice would benefit from the application of systematic optimization techniques.

### 2.1.5. Summary

Despite the lack of adoption of systematic large-scale optimization approaches in practice by transmission planners, the history of formulations and the recent computational advances in integer programming suggest that systematic optimization techniques can potentially be applied successfully to solve realistic optimal transmission expansion problems, with at least a representation of thermal contingency constraints on transmission. The need to plan for transmission expansion to satisfy multiple goals, such as deliverability from multiple renewable resources, suggests that systematic approaches may have considerable benefit compared to *ad hoc* planning, particularly where a large number of potential transmission paths exist such as in the case of the ERCOT CREZ transmission expansion.

We will assume that systematic tools are available to perform transmission planning considering, at least, thermal contingency constraints, but recognize that this is not an innocuous assumption and that issues such as uncertainties in cost estimates and additional constraints can also complicate the planning

process. This issue is discussed further in the next section and in Section 3.7. We assume that an independent entity such as an ISO or Regional Transmission Organization (RTO) performs the transmission planning, but recognize that this will typically also involve input from market participants. The availability of a systematic transmission planning tool will be key to cost allocation.<sup>2</sup>

## *2.2. Transmission construction costs*

### *2.2.1. Economies of scale and lumpiness of new construction*

Transmission services are provided at particular nominal voltage levels. The thermal capacity of the line depends on the product of its voltage level and its current rating. The thermal capacity of lines increases faster than linearly with voltage level because higher voltage lines typically also have higher current ratings, whereas land acquisition and construction costs can increase more slowly. Furthermore, double- or multiple-circuit construction, where two or more transmission lines share towers and “right-of-way,” is generally cheaper per unit capacity than single-circuit construction. Consequently, there are economies of scale in the construction of transmission elements [8, 32, 33]. To take advantage of such economies of scale, typical line additions are in relatively large “lumps” that are often as large or larger than the capacity of large generators (1000 MW) and typically much larger than the capacity of wind turbines (2 MW) and of typical wind farms (100 MW).

### *2.2.2. Other upgrades*

Besides construction of new lines to upgrade capacity, there may be opportunities to upgrade the capacity of specific elements or add new elements to enable an increase in transmission capacity. These upgrades typically enable better use of thermal transmission capacity, allowing operation closer to the thermal contingency limits. Examples of such upgrades include conventional approaches such as reactive compensation [48, 75] as well as more exotic approaches such as “line tensioning devices” [11][35, appendix D]. In some cases, upgrading a circuit breaker can provide for operation of an existing transmission line at a higher thermal rating. We assume that consideration of such upgrades is integrated into the systematic transmission planning process.

### *2.2.3. Estimates*

An essential aspect of systematic transmission expansion planning is the estimation of transmission construction costs, including issues such as land acquisition costs, transmission line construction costs, substation and equipment costs, and the costs of other upgrades. Recent experience in ERCOT, for example, indicates that estimates of transmission construction costs made before construction begins can be subject to considerable error. The ERCOT CREZ transmission optimization study used detailed equipment cost estimates that did not consider regional variations in land acquisition costs [118, §I.D and table 3]. As

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<sup>2</sup>In some cases, a simplified model of transmission planning might be adequate as a basis for the proposed pricing model. For example, the “Transmission Network Use of Services” charge in the United Kingdom is based on a simplified transmission planning model that only considers additions along existing corridors and assumes that transmission is not lumpy [65].

discussed in [97], these “very preliminary cost estimates” for transmission were around \$4.9 billion in 2008 dollars [97, §2.2]. However, the mid-2011 estimate of the project costs had risen to around \$6.8 billion [97], indicating that the preliminary costs considerably underestimated the actual costs in nominal terms and also in real terms. Reasons for the underestimate include assumptions about availability of shortest path rights-of-way between the ends of lines that will actually be constructed as longer lines because of siting difficulties.

Many of the underestimation errors in the CREZ estimates are in principle avoidable, given the large existing amount of data on costs of transmission as embodied in publicly available data, such as FERC form 1 data in the United States [33]. Although there is considerable regional variability in transmission costs and between rural and urban construction, and many specific issues that can affect costs, it seems reasonable that historical costs could be used as a guide to future construction costs, particularly if appropriate construction contingency allowances are included to avoid systematic biases in estimates, and if historical data is used to estimate the ratio of the length of built lines to the distance between the ends of lines. Similar data has been used effectively in the oil and natural gas pipeline industry [23] and historical costs have been used in Brazil to set “reference costs” for competitive procurement of transmission [106, §II.B]. Indeed, systematic transmission optimization is not possible without such estimates in advance of construction. In principle, stochastic programming could be used to represent some of the cost uncertainties.

#### *2.2.4. Efficiency*

Other relevant issues include whether or not individual transmission construction projects are carried out for the lowest construction costs, whether the resulting lines have rated capacities that fully reflect their capabilities, and whether the regulator can observe these costs and capacities, and compare them to the costs, capacities, and implications of upgrading other elements in order to adjudicate whether the most efficient plan has been developed. Incentive structures and performance incentives for transmission firms are discussed in [70, 12, 54, 105]. An approach that has been used in Brazil is to carry out auctions to procure construction of proposed transmission lines [106, §II.A].

#### *2.2.5. Summary*

The lumpiness of transmission construction means that discrete variables are necessary in an optimization formulation. The uncertainty of transmission cost estimates means that there is some inherent uncertainty in the planning process. Moreover, information asymmetries between builders of transmission and the regulator and the ISO complicate the assessment of costs. We will nevertheless assume that usable estimates of construction costs are available, including costs of both line and other upgrades, and that there is a mechanism to provide incentives for efficient planning and construction.

### 2.3. Taxonomy of transmission expansion

We will classify transmission upgrades into “reliability,” “policy,” and “economic” upgrades, partly following the distinctions in [121, 122, 57]. However, we recognize that a particular line may contribute to several categories of upgrades.

#### 2.3.1. Reliability upgrades

Much transmission construction in the United States has historically been built pursuant to so-called “reliability requirements” that seek to ensure that the obligation to serve peak demand can be met under a number of normal and abnormal conditions as specified by the North American Reliability Corporation [81]. Prior to restructuring of the electricity industry, there was typically strong coupling between transmission expansion plans and generation expansion plans, so that these upgrades could be viewed as providing “reliable” transmission from generation to demand.

In the period from the beginning of restructuring in the US in the mid 1990s until relatively recently, however, there has been relatively much less transmission construction than generation construction. Growth in peak demand has been relatively low, while new generation investment has in part replaced retiring or moth-balled generation. Consequently, the focus of “reliability upgrades” has presumably shifted from enabling the integration of *particular* generation resources to meet peak demand forecasts to enabling delivery from the *collection* of available resources to meet peak demand forecasts that grow slowly from year to year [118, page 47]. We will reflect this in some of the specific discussion of policy upgrades, by considering deliverability from a zonal “hub,” rather than from a specific generator, implicitly requiring separate consideration for “inter-regional” transmission upgrades and for new construction to enable delivery to a hub, to be discussed below under “economic” upgrades. This enables us to focus the design of (and charges for) reliability upgrades on well-specified and particular parts of the transmission system, but may require some modifications to the standards for reliability as currently practiced in North America. The optimization formulation for reliability upgrades will be described in Sections 3.3 to 3.5.

#### 2.3.2. Policy upgrades

Many states have built or have plans to build a significant amount of renewable resources. Wind resources in particular have tended to be remote from existing demand centers so that increased integration of such wind resources depends on the construction of inter-regional transmission from wind-rich regions towards demand centers. Typically, the maximum output of wind resources does not coincide with the peak demand conditions, so that upgrades will typically not involve the need to increase deliverability to load, but will instead require upgrades to enable deliverability to a zone or zonal hub in the vicinity of the demand center.

The CREZ transmission construction mentioned in Section 2.2.3 is an example of this type of construction [118, section III], with most of the transmission upgrades taking place outside of demand centers. The CREZ transmission is being built to facilitate policy directions in ERCOT to increase renewable energy.

We will refer to such transmission as “policy upgrades.” Policy upgrades reflect changes in the generation mix that are mandated by policy decisions. The optimization formulation for policy upgrades will be described in Sections 3.3 to 3.5.

### *2.3.3. Economic Upgrades*

“Economic upgrades” include upgrades that will allow delivery of lower cost energy to an area that would otherwise only have available higher cost energy. In particular, this might include “inter-regional” transmission from a low cost to a high cost region, but could also include deliverability from a new generator to a zonal hub. We posit that economic upgrades are desired by market participants who have a willingness-to-pay for these economic upgrades. The upgrades will be requested by market participants through an auction process [35, appendix C]. The auction formulation will be described in Section 3.6.

### *2.3.4. Summary*

Transmission upgrades may be mandated by reliability or policy criteria or be desired by market participants. In Section 3, we will consider how to unify treatment of these upgrades into a single transmission expansion and pricing framework since several categories of upgrades may be accomplished simultaneously and should be priced consistently. Moreover, as articulated in [7], reliability upgrades can be interpreted as a type of economic upgrade where there is a willingness-to-pay for delivery into a load center at a high or infinite price.

## *2.4. Regulatory institutions*

### *2.4.1. Regulated investment*

For most of the model formulations described in Section 2.1, it is tacitly assumed that the planning is undertaken by an agent who is motivated to plan in order to minimize the costs or the costs minus benefits of transmission. In practice, even with the emergence of markets for electricity, the transmission sector is typically structured as a regulated monopoly, or in the case of interconnected regions, as a collection of geographical monopolies, and with transmission planning conducted both by system operators and interested stakeholders, often in conjunction. The continued monopoly status of the transmission and distribution sector is based on several issues, including the “public goods” nature of electricity supply, the necessity to obtain rights-of-way for new transmission, and economies of scale in transmission construction, which imply that competing providers of “parallel” transmission services would be more expensive than a single monopoly provider.<sup>3</sup>

In North America, transmission plans that are approved by the regulator are built by the monopoly transmission provider, the resulting costs added to the “ratebase,” and the investor is awarded a regulated

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<sup>3</sup>Furthermore, to the extent that there are various unpriced benefits of transmission, charging for transmission services in a “competitive” manner that does not reflect all of these benefits would result in remuneration falling short of costs [91].

rate-of-return typically based on the *ex post* construction costs. The rate-of-return in the United States is often around 9%. Although we will discuss property rights in another context below, transmission providers can be viewed as receiving a “property right” for building transmission that could be characterized as a right to a relatively stable income stream in return for providing that transmission for use in the electricity system.<sup>4</sup>

As is well-known, the incentives due to rate-of-return regulation are to over-build relative to minimum requirements [3], exacerbating the concerns over efficient construction described in Section 2.2.4. Particularly given that returns of 9% currently far exceed typical returns of at-risk investments and of US treasury bonds, it could be expected that significant transmission over-building would have taken place recently. Ironically, the expectation and advent of electricity market restructuring in North America was associated with a prolonged period of relatively little transmission being built, as noted above, and transmission construction still appears to be below even the levels needed to replace aging infrastructure [2].

As well as the awarded rate-of-return on regulated investments, a related concern is the assumed discount rate and time horizon for assessing investments. For example, [35, page 16] suggests a much lower “social discount” rate for evaluating transmission projects.

#### 2.4.2. Cost allocation

The cost of “radial” connections from new generators to the existing transmission network are typically allocated to those generators because of the clear connection between this type of transmission construction and its beneficiaries. In addition, when further “network upgrades” are required for deliverability to the load, “deep network charges” may also be allocated to new generation in some jurisdictions.

Costs of transmission that are not specifically allocated to particular generators are typically allocated to the retail customers in the franchise area of a regulated transmission monopoly on a “load-weighted average” or peak load basis to various “customer classes” [49, 7]. That is, a fixed tax per unit energy or per unit peak load is effectively levied within each customer class, with different classes differentiated on the basis of delivery voltage and institutional arrangements, and the total tax chosen to recover the regulated rate-of-return and other approved costs.

In some cases, different criteria are used to allocate the costs rather than a load-weighted average or peak load basis. For example, criteria such as calculations of the contributions to flows on individual lines [83, 117], and assessments based on the benefits [115] have been proposed or used to allocate costs.

Our approach to allocating costs will aim at associating the cost allocation to the beneficiaries of the investment. The next section will discuss benefits assessment.

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<sup>4</sup>An alternative to a fixed rate-of-return is the use of “RPI-X” regulation, where an inflation index is used as a basis for returns. An alternative to utilizing *ex post* costs is to auction the right to build transmission lines, as discussed in [106, §II.A].

### *2.4.3. Benefits assessment*

Recently, the FERC has ordered that costs be allocated based on benefits [122]. If costs and benefits can be identified then, in principle, there are standard prescriptions for cost allocation such as using the “core” of the associated game [27] or Aumann-Shapley pricing [61]. Our approach will be somewhat different in that we will allow for “uplift;” that is, we will continue to allow for a tax to recover some of the transmission costs. However, we aim to minimize the fraction of costs recovered by such a tax. To do so, it is necessary to identify, to the extent possible, the relationship between the desired transmission upgrades and the expansion costs associated with those upgrades, even if the transmission expansion encompasses multiple types of upgrades or multiple beneficiaries.

Several authors have proposed approaches that involve evaluating the incremental benefit of the addition of a transmission line, typically by running dispatch models with and without the transmission expansion [42]. In principle, costs can be allocated based on the benefits received, and incentives can be provided for merchant transmission construction. However, besides the conceptual difficulty of defining the “without” case over time as multiple upgrades are completed, a serious practical difficulty with such approaches is that evaluating the counterfactual dispatch costs without the transmission relies on offers to the market that were intended for the “with” case. That is, the actual offers made after transmission upgrades are completed may not reveal relevant information about what would have been offered in the counterfactual “without” case, at least in the case where the addition of the transmission makes a significant change in the prices.

### *2.4.4. Summary*

We will assume that regulatory institutions are in place to motivate efficient planning of the system and seek to allocate costs to beneficiaries, to the extent possible, according to the drivers of transmission upgrades. However, we will avoid approaches that require comparison of “with” and “without” transmission cases to evaluate the benefits.

## *2.5. Property rights*

Well-defined prices for a service or product require a well-defined definition of that service, and well-defined property rights. This section considers the property rights definition for transmission services.

### *2.5.1. Financial transmission rights*

In restructured electricity markets, rights to transmission are typically based on FTRs. Incremental FTRs are made possible by incremental transmission construction and such incremental rights have been proposed as property rights to be conferred on the builders of new capacity [21, 92, 60, 56, 51, 66, 105]. Incremental FTRs confer the right to the congestion rent associated with an increase in point-to-point deliverability of power between “injection” and “withdrawal” buses made possible by the transmission upgrade.

Property rights based on incremental FTRs allocate the congestion rent in a particular way based on

the increase in point-to-point deliverability.<sup>5</sup> When the issues of lumpiness and economies of scale are significant, however, the building of transmission will typically lead to depressed priced differences between the injection and withdrawal of lines. Moreover, reliability mandates may require increases in transmission capacity to allow for deliverability under extreme events that further depress price differences under normal operation. Historically, estimates of congestion rent have fallen below regulated transmission costs [91, 108]. Consequently, the value of property rights based on FTRs after construction may be below the cost of construction, despite the transmission having net benefits [57]. In such contexts, it is unlikely that market participants would choose to pay for transmission construction costs in order to receive FTRs, unless they can simultaneously acquire contracts to energy at the pre-transmission expansion price. This is exacerbated by “free-rider” implications since other market participants will then also benefit from the lowered price difference. In short, it may be in no market participant’s interest to bid for the construction of such transmission.

Nevertheless, we do not wish to rule out the possibility of cases where the FTRs confer value that covers the transmission construction costs. A particular case where FTRs may be viable is in connecting a new generator to a zonal hub, or other trading point. If the trading point is sufficiently liquid then the addition of the new supply will not significantly change the prices at the trading point. Moreover, the generator investor can internalize the costs of generator construction and the cost of transmission necessary to deliver to the trading point. This situation is clearest in a new radial connection to the trading point.

For simplicity, our focus will be on *obligation* FTRs where the “payoff” from the FTR is negative if the price at withdrawal bus is lower than at the injection bus. Moreover, we will neglect seasonality of FTRs. In principle, the formulation could be expanded to consider *option* FTRs and to consider different seasons.

### 2.5.2. *Compulsion*

In cases where lumpiness of transmission construction, non-excludability, and reliability mandates are likely to lead to significantly depressed values for financial rights after transmission is built, and where forward energy contracts are insufficient to mitigate this issue, it cannot be expected that market participants will find it in their interest to receive FTRs in return for the construction cost of transmission upgrades. This is likely to be the case for both reliability upgrades and policy upgrades and may be an issue in large economic upgrades as well. We will treat these as cases where exogenous policy dictates the increase in desired transfer capability, and then seek consistent prices that compulsorily allocate the cost of that capacity. The parties required to purchase the rights would receive the payoffs from the associated FTRs, but this would not fully compensate them for the compulsory payments. In the case of reliability upgrades, the load is presumably

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<sup>5</sup>Other rights based on congestion rents include “flowgate rights” [25], where rights to the congestion rent are associated to lines with binding transmission constraints, and “border flow rights” [6], where rights to the congestion rent are associated to all individual lines based on the power flow and locational price difference between the ends of the line.



receiving other benefits associated with deliverability under extreme system conditions and in the case of policy upgrades it is presumably receiving benefits due to, for example, lower environmental burden of using renewables.

Such compulsion places a special obligation on the regulator to ensure that transmission plans and construction are carried out efficiently.<sup>6</sup> Although we will not discuss this issue in detail, references [54, 104, 105] suggest that incentive mechanisms for efficient construction are compatible with FTRs, albeit under a different regime to the one we describe. Compulsion also requires careful design to avoid situations where costs can be shifted from one party to another [57, page 11]. We observe that the effort to avoid socialization and to separate out as much as possible the costs of reliability and policy upgrades will help to focus scrutiny on the transmission planning process.

### *2.5.3. Summary*

We will follow [104, 105] in focusing on transmission capability represented in terms of issued FTRs as the property right. Similarly to [36], we will consider the transmission expansion necessary to support incremental FTR requirements. We will allow for voluntary investment by market participants in economic upgrades. However, we will also consider cases where specific mandated investments are funded by compulsory charges to users of transmission based on assessments of reliability needs or policy. We assume that appropriate incentives are in place for efficient planning and construction of transmission upgrades generally, particularly including reliability and policy upgrades.

## *2.6. Uplift in electricity markets*

### *2.6.1. Locational marginal pricing*

Despite the convex approximation to operations that are typically used in planning models, all day-ahead markets and some real-time markets in North America utilize some form of a mixed-integer programming formulation reflecting the discrete and continuous variables in unit commitment and dispatch. The duality gap between the solution of the primal commitment and dispatch problem and the solution of the dual problem obtained by relaxing supply-demand balance at each bus means that typically there are no energy prices that “support” the primal optimal commitment and dispatch solution. That is, there are no energy prices such that all market participants are at least as well off by following the optimal commitment and dispatch as they would be under every other commitment and dispatch [73].

Most day-ahead markets in North America use an approach to this issue that solves for the locational marginal prices, given that the discrete variables are fixed at their optimal values, and then creates a “make-whole” payment to compensate market participants for any opportunity costs of committing and dispatching consistent with the optimal commitment and dispatch. The make-whole payment necessitates

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<sup>6</sup>It should be understood that compulsion is the norm currently for much transmission in North America.

a side-payment, which is universally charged to load as a socialized charge and is called an “uplift.” This approach is analyzed in [86].

### *2.6.2. Extended locational marginal pricing*

An alternative approach is to dualize the supply-demand balance constraints, set “extended locational marginal prices” based on the dual maximizer, and again provide make-whole payments based on compensating the opportunity costs for market participants to commit and dispatch consistently with the optimal solution. This approach is analyzed in [50, 58, 74] and is being considered for implementation at the Midwest ISO. By definition, the use of the dual to set energy prices will minimize the uplift over choices of energy prices that depend only on location.

### *2.6.3. Summary*

Because of the integer decisions in electricity markets, it is the case that no set of prices will “support” optimal commitment and dispatch. Because of the integer decisions in transmission planning, it is similarly the case that no set of prices will “support” the optimal transmission expansion plan. Analogously to the approach developed in [50] for extended locational marginal prices, we will define a suitable dual problem in Section 3.4 that provides prices that minimize uplift of transmission construction costs.

### *2.7. Overall summary*

We summarize the interaction of the type of upgrade, property rights based on FTRs, and cost allocation. First focusing on the requirements for upgrade of thermal capacity to meet reliability requirements, consider upgrades that are needed to deliver power to a load center based on a demand forecast for a future test year. Our approach to incorporating this requirement in an optimization framework and cost allocation mechanism is to consider a collection of supply buses and the collection of load delivery points. For example, a typical arrangement in large cities is to have a high voltage, for example 345 kV, ring built around the city. Necessary upgrades to support delivery of power to load delivery points in the city could be represented by an incremental FTR with injection at a weighted average of the buses in the ring and withdrawals at the load buses. Alternatively, an FTR could have injection at a zonal hub or other trading point.

For example, Figure 1 shows four buses, 1, 2, 3, 4, as large bullets. These four buses are interconnected in a ring by four pairs of double-circuit lines. In addition, there are five other buses, 5, 6, 7, 8, and 9, shown as small bullets, that form the load delivery points to distribution substations and are interconnected to buses 1, 2, 3, 4 with lower capacity lines. An incremental FTR with injection at an equally weighted average of buses 1–4 and withdrawal at a forecast load weighted average of buses 5–9 could be used to represent the requirements for delivery to forecast load. The incremental FTR requirement would be set equal to the forecast load increase over the planning and construction horizon, less any committed supply increase in the city over the horizon. Possession of the FTR would expose loads (or load serving entities) to the hub price. This would have a similar effect to current practices in several jurisdictions of using geographically averaged

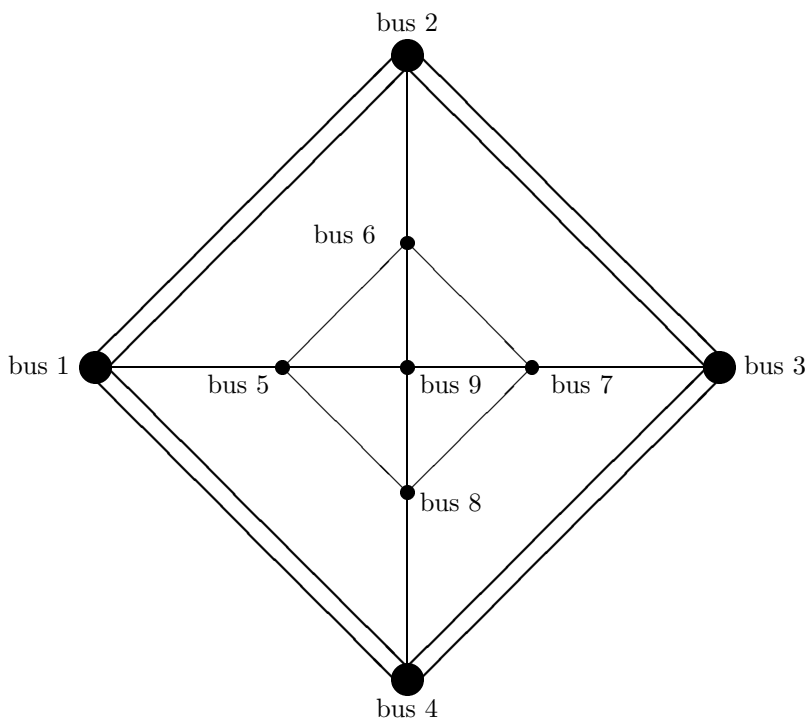


Figure 1: Illustrating FTR for reliability upgrade.

prices for loads, but would preserve incentives at the margin for efficient consumption based on the value of the locational marginal price.

As discussed in Section 2.3.1, we assume that there is a partial separation of planning for increases in urban demand from planning for inter-regional transmission capacity. Consequently, delivery from generation to the ring could be accomplished with a separate FTR specification, either involving a policy upgrade, for example for mandated renewable portfolio standards, or an economic upgrade. For example, for a policy upgrade for wind, a suitable FTR would be from the various potential sources of remote wind power to a zonal hub or a city ring, such as buses 1–4 in Figure 1. Deliverability from remote wind zones towards demand centers was the goal of the CREZ transmission study [118].

To side-step the difficult issue of estimating the benefits of incremental transmission, we will emphasize the need to elicit willingness-to-pay for economic upgrades such as for delivery from another zone or hub to buses 1, 2, 3, and 4, rather than use estimates of prospective benefits developed, for example, as part of a regulatory proceeding. However, we recognize that to the extent that new transmission depresses price differences significantly, economic upgrades may not form a large fraction of transmission construction projects.<sup>7</sup>

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<sup>7</sup>As discussed in Section 2.5.1, forward contracting of energy at pre-expansion prices together with economic upgrades may partly alleviate this issue. Furthermore, to the extent that hubs are already liquid trading points, the addition of transmission

Transmission planning would then seek the optimal construction plan to provide for the incremental FTRs to support reliability, policy, and economic requirements. As will be discussed in the rest of the paper, the dual maximizer corresponding to this expansion problem would provide prices for the incremental transmission. Charges for reliability upgrades could then be allocated to the load directly or to the corresponding load serving entity (LSE). The compulsory allocation of costs for reliability upgrades would be restricted to capacity needed for delivery from the city ring to the load delivery points, under the presumption that there is sufficient supply deliverable to the city ring to support the demand.<sup>8</sup> If there were competitive LSEs, then the costs could be allocated amongst them based on the share of load or some other policy directive. Requirements to build for the peak anticipated demand together with an allocation rule would avoid free-rider issues that might otherwise arise with competitive LSEs. For a policy upgrade, a transparent price for the transmission would be available that could be allocated according to, for example, renewables policy. Finally, economic upgrades would also be assessed prices under the same framework.

To summarize, our institutional setting has a transmission investor/builder, receiving some form of regulated return on transmission upgrades approved by a regulator, with the system planned by an ISO/RTO. The regulator sets prices to market participants, who receive the FTRs created by the transmission construction. For reliability and policy upgrades, the market participants are compelled to purchase the FTRs. For economic upgrades, interested market participants purchase FTRs based on their stated willingness-to-pay.

### 3. Formulation

In this section we consider a formulation of the transmission expansion problem that is aimed at abstracting from the formulations discussed in Section 2 in order to price transmission construction to users of transmission services. Assumptions will be outlined in Section 3.1. The basic network model will then be formulated in Section 3.2. The transmission expansion development is most straightforward in the case of reliability and policy upgrades to support a given desired transfer capability specified by desired incremental FTRs, so we will consider this setting first in Sections 3.3, 3.4, and 3.5. We will then consider the formulation for economic upgrades having an explicit willingness-to-pay in Section 3.6.

#### 3.1. Assumptions

As outlined in Section 2.1.5, we assume that transmission expansion is planned based on the solution to an optimization formulation of the transmission expansion problem. Our formulation is similar to that in [9, 54]. We will assume that some technique is available to solve the optimization problems that we formulate; however, we recognize that these are non-trivial problems to solve.

The objective of this formulation is either to:

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to and from hubs will affect prices there relatively less than, for example, the effect on prices at load delivery points.

<sup>8</sup>If there were inadequate transmission capacity for delivery to the ring then this might be remedied as part of additional reliability upgrades.

- minimize the costs of a desired increase in transmission capability represented in terms of desired incremental FTRs, or
- minimize the costs minus the benefits of incremental transmission expansion, with benefits represented in terms of desired incremental FTRs and associated willingness-to-pay,

or a combination of these objectives. We will assume that total transmission expansion costs are primarily driven by the need to satisfy first-contingency thermal limits [62] and therefore depend primarily on land acquisition and transmission line construction costs. Our main focus will be on representing these costs explicitly into the objective of an optimal transmission planning process. We will assume that reasonable estimates of these costs are available from historical data as described in Section 2.2.3.

However, as mentioned in Section 2.1.4, some reliability issues and voltage and stability constraints will typically necessitate additional upgrades over and above those required to satisfy first-contingency thermal constraints. Moreover, sharing of generation reserves requires that sufficient unloaded transmission capacity is available and issues such as improving the competitiveness of the energy market may also require additional transmission [22]. To the extent that these additional issues necessitate additional upgrades to achieve a given transfer level or satisfy other requirements, these constraints will be handled outside of the main optimization process, although we tacitly assume that there would also be an optimization process involved in choosing these additional upgrades. We assume that these additional costs are allocated to load or load serving entities in a separate process, to be discussed in Section 3.7.

We also tacitly assume that:

- there is a fairly large set of candidate lines and other upgrades that could be built [122, ¶38];
- the costs of each candidate upgrade are well-characterized; and
- the goal is to assemble an incremental portfolio of a subset of these candidate lines and other upgrades to achieve the objective.

### 3.2. Basic network model

We will represent the existing system and existing allocation of FTRs in terms of:

- existing line and other element capacities specified by the vector  $k \in \mathbb{R}_+^{n_k}$ ;
- an existing bus admittance matrix  $Y \in \mathbb{R}^{n_b \times n_b}$ ;
- existing FTRs specified by  $f \in \mathbb{R}^{n_f}$ ;
- net injections  $y \in \mathbb{R}^{n_b}$  at all buses except the reference bus for the implied dispatch corresponding to  $f$ ; and
- net injections–FTR incidence matrix  $A \in \mathbb{R}^{n_b \times n_f}$ .

where:

- $n_k$  is the number of existing and possible new transmission lines;
- $n_b$  is the number of existing and possible new buses, excluding the reference bus; and,
- $n_f$  is the number of existing and possible new FTRs.<sup>9</sup>

The relationship between  $f$ ,  $A$ , and  $y$  is that if FTR  $\ell$  is for injection at bus  $r$  and withdrawal at bus  $s$  then the  $\ell$ -th column of  $A$  has:

- (if  $r$  is not the reference bus) a 1 in its  $r$ -th entry, and
- (if  $s$  is not the reference bus) a  $-1$  in its  $s$ -th entry.

Consequently, the net injections and FTRs are related by:

$$y = Af.$$

If a line currently exists or a particular FTR is currently issued, then the corresponding entry of  $k$  or  $f$  is non-zero. However, if a particular pair of buses is not currently joined by a line then the corresponding entry of  $k$  is zero. Similarly, we may be considering the possibility of creating FTRs between buses in cases where no FTRs have previously been issued between those buses. The corresponding entry of  $f$  for the existing system would be zero.

Let  $p : \mathbb{R}^{n_b \times n_b} \times \mathbb{R}^{n_b} \rightarrow \mathbb{R}^{n_k}$  represent the flows on the lines in the system. In particular,  $p(Y, y)$  is the vector of flows on the lines given existing admittances  $Y$  and the implied dispatch  $y$  corresponding to the existing FTRs  $f$ , so that simultaneous feasibility of FTRs issued in the existing system is represented by:

$$p(Y, y) \leq k,$$

assuming one-to-one correspondence between constraints and line capacities. We assume that simultaneous feasibility is satisfied for the existing FTRs in the existing system. If simultaneous feasibility is satisfied, then the congestion rent is adequate to cover the FTR obligations [55].

The notation is most easily interpreted in terms of pre-contingency thermal limits on transmission. However, with a more elaborate interpretation, or some complication of the notation, these simultaneous

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<sup>9</sup>There is the issue that the existing system may not be fully subscribed [67]. In this case, it may be appropriate to auction, using a conventional FTR auction, the remaining existing capacity, obtaining an initial set of supported FTRs,  $f$ , that more fully utilizes the existing capacity. The issue of timing between auctioning of the existing system and building new transmission capacity is not explicitly considered in this paper, although this will in practice be a significant issue that will complicate the allocation of existing and future capacity. Moreover, we do not consider the implications of paying for transmission expansion as a capital expenditure versus “ratebasing” the expenditure and charging transmission customers an annualized contribution to the ratebase.

feasibility constraints could represent contingency constraints on transmission and more complicated representation of constraints, including requirements for deliverability under multiple outages. Some of the examples in Section 4 will illustrate more complex representations.

Furthermore, although the symbols suggest an emphasis on real power, the analysis could also include other types of constraints that can be represented in terms of line and other element capacities. For example, constraints on line current or on apparent power could be accommodated. Moreover, FTRs can be issued for specific seasons and with optionality [84]. For simplicity, however, we will not explicitly develop notation to consider these cases of constraints in terms of current or apparent power, option rights, or seasonality.

The notation does accommodate an AC power flow formulation [128]. However, practical solution is likely to require the use of the DC power flow approximation [128, section 4.1.4]. The examples in Section 4 will use the DC power flow approximation. In the context of the DC power flow approximation, we can consider the shift factor matrix,  $C : \mathbb{R}^{n_b \times n_b} \rightarrow \mathbb{R}^{n_k} \times \mathbb{R}^{n_b}$ , which specifies the fraction of flow on each line due to injection at a bus and withdrawal at the reference bus. For an implied dispatch  $y$ , the flows on the lines are given by  $C(Y)y$ , so that simultaneous feasibility in the existing system would be represented by:

$$C(Y)y \leq k.$$

### 3.3. Expansion to support specified desired incremental FTRs

Incremental transfer capabilities are specified in terms of desired incremental FTRs, specified by the vector  $\Delta f \in \mathbb{R}^{n_f}$ , that are desired at any price. For example, these incremental FTRs could be chosen to support reliability requirements or for policy upgrades or both. In the particular case that no additional FTRs are required between a particular pair of buses then the corresponding entry of  $\Delta f$  is zero.

To provide for these incremental FTRs, we consider expansion of line and other element capacities by  $\Delta k \in \mathbb{R}^{n_k}$ , chosen from a set  $\Delta K \subset \mathbb{R}^{n_k}$  of possible additions. For completeness, we assume that  $\Delta K$  includes the possibility of no additional line construction, so  $\mathbf{0} \in \Delta K$ . We also assume that the cost of new transmission and the resulting change in admittance of the system are specified by the functions  $c : \mathbb{R}^{n_k} \rightarrow \mathbb{R}$  and  $\Delta Y : \mathbb{R}^{n_k} \rightarrow \mathbb{R}^{n_b \times n_b}$ , respectively, so that the cost of the line and other element expansion is  $c(\Delta k)$ , resulting in a change in the admittance matrix of  $\Delta Y(\Delta k)$ . (In Section 3.7, we will consider additional costs that are not accounted for in the function  $c$ .) Note that  $c(\mathbf{0}) = 0$  and  $\Delta Y(\mathbf{0}) = \mathbf{0}$ .

Note that changes in capacity are explicitly assumed to change the admittance matrix and therefore to change the functional form of  $p$  and (in the DC power flow approximation) change the values in the shift factor matrix  $C$ . This captures an essential aspect of electric transmission that distinguishes it from more conventional transportation networks.

A formal version of the minimum cost transmission expansion problem is then:

$$\min_{\Delta k \in \Delta K} \{c(\Delta k) | p(Y + \Delta Y(\Delta k), y + A\Delta f) \leq k + \Delta k\}. \quad (1)$$

A version based on DC power flow is:

$$\min_{\Delta k \in \Delta K} \{c(\Delta k) | C(Y + \Delta Y(\Delta k))(y + A\Delta f) \leq k + \Delta k\}.$$

Write  $\Delta k^*$  for the minimizer of problem (1).

### 3.4. Prices for incremental FTRs

The approach we propose for allocating costs of construction to incremental FTRs is analogous to the approaches developed in [50] for finding prices in electricity markets that minimize the uplift. In the case of pricing transmission, we will incorporate a new vector variable into the formulation in (1) that represents the actual change in simultaneously feasible implied dispatch and also add constraints to (1) to require that the actual change in simultaneously feasible implied dispatch equals the change necessary to support the desired FTRs. Finally, we dualize these constraints and set prices based on the maximizer of the dual problem.

In particular, we define a new variable  $\Delta y \in \mathbb{R}^{n_b}$  that represents the actual change in simultaneously feasible implied dispatch made possible by the incremental transmission, and add a constraint to require  $\Delta y = A\Delta f$ , where we recall that  $\Delta f$  is the vector of desired incremental FTRs. Let  $\delta \in \mathbb{R}_+$  be a parameter. Then consider the following problem that is equivalent to (1):

$$\min_{\substack{\Delta k \in \Delta K, \\ \Delta y}} \{c(\Delta k) | p(Y + \Delta Y(\Delta k), y + \Delta y) \leq k + \Delta k, \Delta y = A\Delta f, \Delta y \leq A\Delta f(1 + \delta)\}. \quad (2)$$

Note that the minimizer of problem (2) is given by  $\Delta k^*$  and  $\Delta y^* = A\Delta f$ . Also note that we have included a redundant constraint  $\Delta y \leq A\Delta f(1 + \delta)$  whose significance will become clear in the context of the dual.

We now consider the dual of problem (2) obtained by dualizing the constraint  $\Delta y = A\Delta f$ . (We do not dualize the constraint  $\Delta y \leq A\Delta f(1 + \delta)$ .) The resulting multipliers from the optimal dual will be used to set prices for the incremental FTRs. The dual problem is:

$$\sup_{\lambda} \inf_{\substack{\Delta k \in \Delta K, \\ \Delta y}} \left\{ \begin{array}{l} c(\Delta k) \\ -\lambda^\dagger(\Delta y - A\Delta f) \end{array} \left| \begin{array}{l} p(Y + \Delta Y(\Delta k), y + \Delta y) \leq k + \Delta k, \\ \Delta y \leq A\Delta f(1 + \delta) \end{array} \right. \right\}. \quad (3)$$

We assume that problem (3) has a maximizer  $\lambda^{**}$  and that the inner problem in (3) has a minimizer corresponding to  $\lambda^{**}$ . Let  $\Delta k^{**}$  and  $\Delta y^{**}$  be the minimizer of the inner problem evaluated for  $\lambda = \lambda^{**}$ . The constraint  $\Delta y \leq A\Delta f(1 + \delta)$  effectively decreases the duality gap and will limit the amount of transmission costs that are to be socialized.

Analogously to the discussion in [50], the transmission plan would be to build the optimal amount of transmission  $\Delta k^*$  from the solution of problem (1), but then charge for the so-created incremental FTRs at the prices implied by the solution of problem (3). In particular, we define the vector of prices for the FTRs  $\Delta f$  to be:

$$\rho^{**} = A^\dagger \lambda^{**}. \quad (4)$$



Using the prices  $\rho^{**}$ , the charges for the FTRs will provide a remuneration for transmission construction equal to:

$$R = [\rho^{**}]^\dagger \Delta f = [\lambda^{**}]^\dagger A \Delta f,$$

which can, in general, fall short of the cost of construction  $c(\Delta k^*)$ .<sup>10</sup> However, we can bound the shortfall in terms of the duality gap between problems (2) and (3) as follows.

Because of the duality gap between (2) and (3), we have that:

$$c(\Delta k^*) = \gamma + c(\Delta k^{**}) - [\lambda^{**}]^\dagger (\Delta y^{**} - A \Delta f), \quad (5)$$

where  $\gamma \geq 0$  is the duality gap. Note that since  $\lambda^{**}$  maximizes the dual then, over choices of  $\lambda$ , the value of  $\gamma$  is minimized by  $\lambda^{**}$ . For large-scale problems we can expect that  $\gamma$  will be relatively small.

Note that  $\Delta k = \mathbf{0}$  and  $\Delta y = \mathbf{0}$  is feasible for the inner problem. Consequently, by definition of the minimizing value  $\Delta k^{**}$  and  $\Delta y^{**}$  in the inner problem in (3),

$$\begin{aligned} & c(\Delta k^{**}) - [\lambda^{**}]^\dagger (\Delta y^{**} - A \Delta f) \\ &= \min_{\substack{\Delta k \in \Delta K, \\ \Delta y}} \left\{ \begin{array}{l} c(\Delta k) \\ - [\lambda^{**}]^\dagger (\Delta y - A \Delta f) \end{array} \left| \begin{array}{l} p(Y + \Delta Y(\Delta k), y + \Delta y) \leq k + \Delta k, \\ \Delta y \leq A \Delta f (1 + \delta) \end{array} \right. \right\}, \\ &\leq c(\mathbf{0}) - [\lambda^{**}]^\dagger (\mathbf{0} - A \Delta f). \end{aligned}$$

Subtracting  $[\lambda^{**}]^\dagger A \Delta f$  from both sides of this inequality, and noting that  $c(\mathbf{0}) = 0$ , we obtain that:

$$0 = c(\mathbf{0}) - [\lambda^{**}]^\dagger \mathbf{0} \geq c(\Delta k^{**}) - [\lambda^{**}]^\dagger \Delta y^{**}. \quad (6)$$

Using the definition (4) of the prices  $\rho^{**}$  and remuneration  $R$ , re-arranging (5), and substituting from (6), we obtain:

$$\begin{aligned} c(\Delta k^*) - R &= c(\Delta k^*) - [\lambda^{**}]^\dagger A \Delta f, \\ &= \gamma + c(\Delta k^{**}) - [\lambda^{**}]^\dagger \Delta y^{**}, \\ &\leq \gamma. \end{aligned}$$

That is, the shortfall between the remuneration for FTRs based on prices  $\rho^{**}$  and the estimated cost  $c(\Delta k^*)$  of the transmission expansion is bounded by the duality gap  $\gamma$ . This is analogous to similar results in the context of electricity markets [50]. The shortfall, together with any other costs not explicitly accounted for in the model of costs  $c$ , would be uplifted to load. Indeed,  $\lambda^{**}$  is the set of prices that minimize  $\gamma$  and so are the “best” linear prices for allocating as much as possible of the cost of transmission expansion using linear prices that depend only on the location of the points of injection and withdrawal for the incremental FTRs.

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<sup>10</sup>Even if only a sub-optimal transmission plan can be found because of computational limitations, the approach can be used to find prices that minimize uplift required for the actually constructed plan. This is analogous to the discussion in [50, pages 29–31].

### 3.5. Role of $\delta$

The constraint  $\Delta y \leq A\Delta f(1+\delta)$  is a ‘‘cut’’ that reduces the duality gap of the dual problem and therefore tends to reduce the amount of construction costs that must be socialized. For example, for  $\delta$  very small, effectively all of the costs of construction will be allocated to the FTRs and there will be no socialized costs. The drawback of this constraint is that it makes the prices non-monotonic in the desired FTR quantities. It is also important to note that this device does not help with solving the dual problem, since it re-introduces a constraint that is as difficult to represent in the inner problem as the constraint that has been dualized

Although we have defined a single parameter  $\delta$  that applies to all constraints, this could be generalized. For example, tighter constraints could be applied for radial interconnections than for other lines under the assumption that radial interconnections should be paid primarily by the interconnecting entity. That is, policy preferences regarding socialization can be implemented through choice of  $\delta$ .

### 3.6. Bids for incremental transmission

To supplement consideration of reliability and policy upgrades treated in previous sections, we will treat economic upgrades in this section. In particular, we consider eliciting willingness-to-pay for incremental transmission in the form of bids (maximum quantities and bid prices). As previously, we will still assume that FTRs to support reliability and policy upgrades are specified as desired incremental FTRs  $\Delta f \in \mathbb{R}^{n_f}$  having no (explicit) willingness-to-pay; however, we also assume that desired FTRs  $\Delta \phi \in \mathbb{R}^{n_f}$  to support economic upgrades are specified with an explicit (maximum) willingness-to-pay. To simplify the discussion, we assume that the latter desired FTRs are each represented in terms of a single bid price up to a maximum desired quantity. In particular, let  $\rho$  be a vector of bid prices and let  $\overline{\Delta \phi}$  be a vector of corresponding maximum desired quantities of FTRs at these prices.

The version of the problem including bids for FTRs is:

$$\min_{\substack{\Delta k \in \Delta K, \\ \Delta y, \\ \Delta \phi}} \left\{ c(\Delta k) - \rho^\dagger \Delta \phi \left| \begin{array}{l} p(Y + \Delta Y(\Delta k), y + \Delta y) \leq k + \Delta k, \\ \Delta y = A(\Delta f + \Delta \phi), \Delta y \leq A(\Delta f + \Delta \phi)(1 + \delta), \\ \mathbf{0} \leq \Delta \phi \leq \overline{\Delta \phi} \end{array} \right. \right\}. \quad (7)$$

Let  $\Delta k^*$ ,  $\Delta y^*$ , and  $\Delta \phi^*$  be the minimizer of this problem. (Note that  $\Delta y^* = A(\Delta f + \Delta \phi^*)$ .) This would result in allocations of FTRs,  $\Delta f$  and  $\Delta \phi^*$ , and a corresponding transmission expansion plan  $\Delta k^*$ .

We propose that the prices for this allocation are derived from the solution of the following problem, which is analogous to problem (3), and is obtained from problem (7) by dualizing the constraint  $\Delta y = A(\Delta f + \Delta \phi)$ :

$$\sup_{\lambda} \inf_{\substack{\Delta k \in \Delta K, \\ \Delta y, \\ \Delta \phi}} \left\{ c(\Delta k) - \rho^\dagger \Delta \phi - \lambda^\dagger (\Delta y - A(\Delta f + \Delta \phi)) \left| \begin{array}{l} p(Y + \Delta Y(\Delta k), y + \Delta y) \leq k + \Delta k, \\ \Delta y \leq A(\Delta f + \Delta \phi)(1 + \delta), \\ \mathbf{0} \leq \Delta \phi \leq \overline{\Delta \phi} \end{array} \right. \right\}. \quad (8)$$

We assume that problem (8) has a maximizer  $\lambda^{**}$  and that the inner problem in (8) has a minimizer corresponding to  $\lambda^{**}$ . Let  $\Delta k^{**}$ ,  $\Delta y^{**}$ , and  $\Delta \phi^{**}$  be the minimizer of the inner problem evaluated for  $\lambda = \lambda^{**}$ .

The transmission plan would again be to build the optimal amount of transmission  $\Delta k^*$  from the solution of problem (7), but then charge for the so-created incremental FTRs at prices based on the solution of problem (8). An issue that arises is that the prices  $A^\dagger \lambda^{**}$  may exceed the bid prices for some of the incremental FTRs. That is, the prices  $A^\dagger \lambda^{**}$  are not market clearing.

Instead of pricing all incremental FTRs based on  $A^\dagger \lambda^{**}$ , we propose a pricing rule that ensures that no issued FTRs are priced higher than the corresponding willingness-to-pay and then show that the shortfall between the payment based on this pricing rule and the estimated cost is still bounded by the duality gap  $\gamma$ . Although this approach has the drawback of resulting in discriminatory prices, which are pay-as-bid in some cases, we will see that it has the significant advantage that the payment rule “supports” the optimal FTRs  $\Delta f$  and  $\Delta \phi^*$ . That is, it provides a unified approach to allocating transmission costs for reliability, policy, and economic upgrades.

First define  $\rho^{**} = A^\dagger \lambda^{**}$  as previously. The incremental FTRs  $\Delta f$  are charged this price, so that the remuneration for these FTRs is  $[\rho^{**}]^\dagger \Delta f$ .

For the FTRs bid with a willingness-to-pay  $\rho$ , define the following partition of  $\Phi = \{1, \dots, n_f\}$ :

$$\begin{aligned}\Phi_{0\leq} &= \{\ell \in \Phi | \Delta \phi^* = 0 \text{ and } \rho_\ell \leq \rho_\ell^{**}\}, \\ \Phi_{0>} &= \{\ell \in \Phi | \Delta \phi^* = 0 \text{ and } \rho_\ell > \rho_\ell^{**}\}, \\ \Phi_{\overline{\Delta \phi} <} &= \{\ell \in \Phi | \Delta \phi^* = \overline{\Delta \phi} \text{ and } \rho_\ell < \rho_\ell^{**}\}, \\ \Phi_{\overline{\Delta \phi} \geq} &= \{\ell \in \Phi | \Delta \phi^* = \overline{\Delta \phi} \text{ and } \rho_\ell \geq \rho_\ell^{**}\}.\end{aligned}$$

Note that for  $\ell \in \Phi_{0\leq} \cup \Phi_{\overline{\Delta \phi} \geq}$ , the price  $\rho_\ell^{**}$  “supports” the optimal incremental FTRs  $\Delta \phi_\ell^*$  in that, given the bid willingness-to-pay  $\rho_\ell$  and the price  $\rho_\ell^{**}$ , the bidder would not prefer to purchase an amount of FTRs that is different to the optimal incremental FTRs  $\Delta \phi_\ell^*$ . On the other hand, for  $\ell \in \Phi_{0>} \cup \Phi_{\overline{\Delta \phi} <}$ , the price  $\rho_\ell^{**}$  does not support the optimal incremental FTRs  $\Delta \phi_\ell^*$  in that, given the bid willingness-to-pay  $\rho_\ell$  and the price  $\rho_\ell^{**}$ , the bidder would not choose to purchase  $\Delta \phi_\ell^*$  of incremental FTRs. For  $\ell \in \Phi_{0>} \cup \Phi_{\overline{\Delta \phi} <}$ , we propose a “make-whole” payment that induces behavior consistent with  $\Delta \phi_\ell^*$ . In the case of  $\ell \in \Phi_{0>}$  the make-whole payment is only necessary if the bidder can require payment of the opportunity cost of not being awarded the FTRs at the price  $\rho_\ell^{**}$ .<sup>11</sup> This is reflected in the alternatives below for this case.

Define the charges for the incremental FTRs as follows:

$\forall \ell \in \Phi_{0\leq} \cup \Phi_{\overline{\Delta \phi} \geq}$ , define the price charged for incremental FTRs  $\Delta \phi_\ell^*$  to be  $\rho_\ell^{**}$ , resulting in remuneration for transmission construction of  $\rho_\ell^{**} \Delta \phi_\ell^*$ ,

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<sup>11</sup>This is analogous to a payment for being “constrained off” in a unit commitment setting [50, page 12].

$\forall \ell \in \Phi_{0>}$ , do not award incremental FTRs and either:

- do not compensate the bidder, assuming compensation of opportunity costs is not required, resulting in remuneration for transmission construction of zero, or
- pay the bidder the opportunity cost  $(\rho_\ell - \rho_\ell^{**})\overline{\Delta\phi}_\ell$ ,

with (non-positive) remuneration for transmission construction bounded below by  $-(\rho_\ell - \rho_\ell^{**})\overline{\Delta\phi}_\ell$ ,

$\forall \ell \in \Phi_{\overline{\Delta\phi}<}$ , define the price charged for incremental FTRs  $\Delta\phi_\ell^*$  to be  $\rho_\ell$ , resulting in remuneration  $\rho_\ell\Delta\phi_\ell^*$ .<sup>12</sup>

Adding together the net remunerations for the FTRs  $\Delta\phi^*$  and the remuneration for the FTRs  $\Delta f$ , the total net remuneration is at least  $R$ , where:

$$R = \sum_{\ell \in \Phi_{0\leq} \cup \Phi_{\overline{\Delta\phi}\geq}} \rho_\ell^{**}\Delta\phi_\ell^* - \sum_{\ell \in \Phi_{0>}} (\rho_\ell - \rho_\ell^{**})\overline{\Delta\phi}_\ell + \sum_{\ell \in \Phi_{\overline{\Delta\phi}<}} \rho_\ell\Delta\phi_\ell^* + [\rho^{**}]^\dagger \Delta f. \quad (9)$$

Again, this remuneration can fall short of the cost of the transmission construction  $c(\Delta k^*)$ . However, we again show that the shortfall is bounded by the duality gap of problem. Paralleling the earlier argument, because of the duality gap  $\gamma$  between (7) and (8), we have that:

$$c(\Delta k^*) - \rho^\dagger \Delta\phi^* = \gamma + c(\Delta k^{**}) - \rho^\dagger \Delta\phi^{**} - [\lambda^{**}]^\dagger (\Delta y^{**} - A(\Delta f + \Delta\phi^{**})). \quad (10)$$

Again observe that the cost of zero transmission expansion is zero, so that  $c(\mathbf{0}) = 0$ . Moreover,  $\Delta k = \mathbf{0}$

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<sup>12</sup>In some cases, a bid with a price  $\rho_\ell$  that is well below  $\rho_\ell^{**}$  might be accepted. To prevent such an event, it may be appropriate to require all bid prices to be at or above minimum reservation prices or use other “activity rules.”

and  $\Delta y = \mathbf{0}$  together with any  $\mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi}$  is feasible for the inner problem in (8). Consequently,

$$\begin{aligned}
& c(\Delta k^{**}) - \rho^\dagger \Delta\phi^{**} - [\lambda^{**}]^\dagger (\Delta y^{**} - A(\Delta f + \Delta\phi^{**})) \\
&= \min_{\substack{\Delta k \in \Delta K, \\ \Delta y \\ \Delta\phi}} \left\{ \begin{array}{l} c(\Delta k) - \rho^\dagger \Delta\phi \\ - [\lambda^{**}]^\dagger (\Delta y - A(\Delta f + \Delta\phi)) \end{array} \left| \begin{array}{l} p(Y + \Delta Y(\Delta k), y + \Delta y) \leq k + \Delta k, \\ \Delta y \leq A(\Delta f + \Delta\phi)(1 + \delta), \\ \mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi} \end{array} \right. \right\} \\
&\leq \min_{\Delta\phi} \left\{ c(\mathbf{0}) - \rho^\dagger \Delta\phi - [\lambda^{**}]^\dagger (\mathbf{0} - A(\Delta f + \Delta\phi)) \mid \mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi} \right\}, \\
&\quad \text{since } \Delta k = \mathbf{0}, \Delta y = \mathbf{0}, \text{ and any } \mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi} \text{ satisfy the constraints} \\
&\quad \text{in the problem in the previous line,} \\
&= \min_{\Delta\phi} \left\{ -(\rho - \rho^{**})^\dagger \Delta\phi + [\rho^{**}]^\dagger \Delta f \mid \mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi} \right\}, \\
&\quad \text{since } c(\mathbf{0}) = 0 \text{ and } \rho^{**} = A^\dagger \lambda^{**}, \\
&= \min_{\Delta\phi} \left\{ -\sum_{\ell \in \Phi} (\rho_\ell - \rho_\ell^{**}) \Delta\phi_\ell + [\rho^{**}]^\dagger \Delta f \mid \mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi} \right\}, \\
&= -\sum_{\ell \in \Phi_{0 \leq}} (\rho_\ell - \rho_\ell^{**}) \times 0 - \sum_{\ell \in \Phi_{0 >}} (\rho_\ell - \rho_\ell^{**}) \overline{\Delta\phi}_\ell \\
&\quad - \sum_{\ell \in \Phi_{\overline{\Delta\phi} <}} (\rho_\ell - \rho_\ell^{**}) \times 0 - \sum_{\ell \in \Phi_{\overline{\Delta\phi} \geq}} (\rho_\ell - \rho_\ell^{**}) \overline{\Delta\phi}_\ell + [\rho^{**}]^\dagger \Delta f, \\
&\quad \text{on evaluating the minimizer of each term in the summation,} \\
&= -\rho^\dagger \Delta\phi^* + \sum_{\ell \in \Phi_{0 \leq} \cup \Phi_{\overline{\Delta\phi} \geq}} \rho_\ell^{**} \Delta\phi_\ell^* - \sum_{\ell \in \Phi_{0 >}} (\rho_\ell - \rho_\ell^{**}) \overline{\Delta\phi}_\ell + \sum_{\ell \in \Phi_{\overline{\Delta\phi} <}} \rho_\ell \Delta\phi_\ell^* + [\rho^{**}]^\dagger \Delta f, \\
&\quad \text{by definition of } \phi^*, \Phi_{0 >}, \text{ and } \Phi_{\overline{\Delta\phi} <}, \\
&= -\rho^\dagger \Delta\phi^* + R,
\end{aligned}$$

where  $R$  was defined in (9). Substituting this bound into the right-hand side of (10), adding  $\rho^\dagger \Delta\phi^*$  to both sides, and rearranging we obtain:

$$c(\Delta k^*) - R \leq \gamma.$$

That is, the total remuneration falls short of the estimated costs of transmission by no more than the duality gap. The prices provide a unified approach to allocation of costs of transmission for reliability, policy, and economic upgrades that minimizes the uplift.

### 3.7. Additional costs

The function  $c$  representing transmission and other element expansion costs should be chosen to include as much of the actual costs of expansion as can be represented as a function of line and other element capacities. This must necessarily be an estimate, and, as noted in Section 2.2.3, could in principle include construction contingency allowances to avoid systematic underestimation biases.

As mentioned in Sections 1 and 2.1.5, however, it is likely that, due to voltage or other constraints [75], and more generally due to “reliability requirements” [81], it is necessary to include a number of additional

upgrades to support the incremental FTRs and that the detailed costs of these upgrades cannot be directly represented in the functional form of  $c$  without significantly complicating the transmission expansion model, since these costs typically depend on issues that are not captured by the incremental thermal capacity of lines.<sup>13</sup> Moreover, uncertainty of future scenarios may motivate the construction of additional lines or upgrades to provide for flexibility, particularly under extreme outage conditions [30, 123].

These additional upgrades result in costs beyond  $c$ . Let  $\Delta c(\Delta k, \Delta y)$  be these additional costs, where we note that they will in general depend on the change in the implied dispatch due to the incremental FTRs as well as on the change in thermal capacity (and possibly on other issues). We propose that these additional costs would be socialized as an “uplift,” as with the allocation of the duality gap.

## 4. Examples

We consider several examples. For simplicity, the DC power flow approximation is used. In Section 4.1, a two bus radial system with contingency constraints is investigated to demonstrate that contingency constraints can be handled within the framework.

In Section 4.2, a looped system is considered. However contingency constraints are ignored for simplicity. Unlike in radial systems, changes in the capacity of corridors of a looped system will typically change the shift factor matrix, which changes both the coefficient matrix and the right-hand side of the constraints. It is important to demonstrate the proposed framework can work in such a context.

These first two examples consider expansion in a single corridor. However, transmission expansion typically involves several upgrades, and expansion to integrate large amounts of renewable resources will involve construction of multiple lines. A three bus radial system with upgrades in two corridors is considered in Section 4.3, again ignoring contingency constraints.

The first three examples involve cases with only limited alternatives for new transmission line construction. In practice, there are typically multiple alternative construction plans that could achieve a required incremental FTR increase. A three bus system that illustrates “network” upgrades with alternative feasible solutions in a stylized manner is considered in Section 4.4.

### 4.1. Two bus system

#### 4.1.1. Existing system

Consider the simple two bus system shown in Figure 2. It has two identical lines joining the two buses, each with 1000 MW of nominal thermal capacity. Bus 2 is the reference bus, so  $n_b = 1$ ,  $A = \begin{bmatrix} 1 & \end{bmatrix}$ , and the vector of net injections is  $y = Af = f \in \mathbb{R}$ .

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<sup>13</sup>Furthering the analogy with models of power system operation, these additional costs could be viewed in the same light as the cost of “reliability unit commitment” [50, pages 30–31].

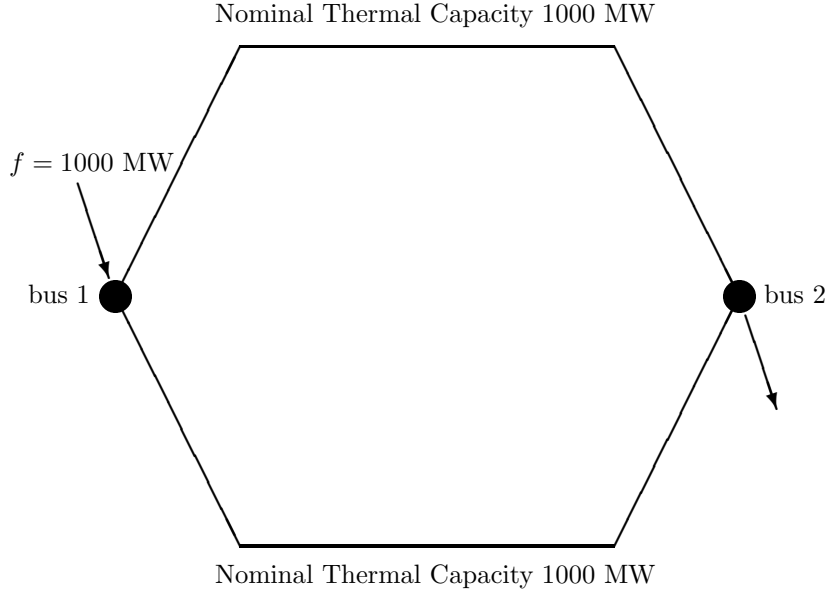


Figure 2: Existing two bus network in example of Section 4.1.1.

If both lines are in-service then, after a contingency of either line, there would be 1000 MW of capacity remaining in the corridor between the buses. Therefore, the first-contingency transmission limit between bus 1 and bus 2 is 1000 MW. We assume that  $f = 1000$  MW of FTRs from bus 1 to bus 2 have been issued on the existing system, as shown in Figure 2, so that the capacity of the existing system from bus 1 to bus 2 is fully subscribed.

We also suppose that capacity can be built in the corridor in 1000 MW increments at a cost of  $\$10^9$  per 1000 MW increment. We assume that the new lines would be identical to the existing lines. Consequently, we can characterize the increased nominal thermal capacity in terms of a non-negative integer variable  $\Delta N$  that specifies the number of additional line increments built. Furthermore, the increase in nominal thermal capacity would equal the increase in first-contingency capacity from bus 1 to bus 2 and therefore equal the increase in FTRs that can be issued.

#### 4.1.2. Specified desired incremental FTRs

Suppose that the desired increase in FTRs from bus 1 to bus 2 is specified by  $\Delta f = 500$  MW. Because of the simple form of the network, the flow between the buses is  $f + \Delta f = 1500$ . Simultaneous feasibility respecting the contingency constraints requires that this flow be no more than  $1000 + 1000 \times \Delta N$  MW, and the cost is  $c(\Delta k) = \$10^9 \times \Delta N$ . Problem (1) is then:

$$\min_{\Delta N \in \mathbb{Z}_+} \{ \$10^9 \times \Delta N \mid 1500 \leq 1000 + 1000 \times \Delta N \}.$$

Table 1: Calculations for Problem (3) from Section 4 under assumption that  $\delta < 1$ .

$\lambda$	$\Delta N^{**}$	$\Delta y^{**}$	$\$10^9 \times \Delta N^{**} - \lambda^\dagger(\Delta y^{**} - 500)$
0	0	0	0
$\$2 \times 10^6/(1 + \delta) - \epsilon$	0	0	$\$10^9/(1 + \delta) - 500 \times \epsilon$
$\$2 \times 10^6/(1 + \delta)$	1	$500(1 + \delta)$	$\$10^9/(1 + \delta)$
$\$2 \times 10^6/(1 + \delta) + \epsilon$	1	$500(1 + \delta)$	$\$10^9/(1 + \delta) - 500 \times \epsilon$

The optimizer of this problem is  $\Delta N^* = 1$ , corresponding to one additional line being built, as shown in Figure 3.

Problem (2) is:

$$\min_{\Delta N \in \mathbb{Z}_+, \Delta y} \{ \$10^9 \times \Delta N \mid 1000 + \Delta y \leq 1000 + 1000 \times \Delta N, \Delta y = 500, \Delta y \leq 500(1 + \delta) \},$$

which has optimizer  $\Delta N^* = 1$  and  $\Delta y^* = 500$  MW.

Problem (3) is:

$$\sup_{\lambda} \inf_{\Delta N \in \mathbb{Z}_+, \Delta y} \{ \$10^9 \times \Delta N - \lambda^\dagger(\Delta y - 500) \mid 1000 + \Delta y \leq 1000 + 1000 \times \Delta N, \Delta y \leq 500(1 + \delta) \}.$$

To obtain the optimizer of this problem, we search over possible values of  $\lambda$  as shown in Table 1, where we let  $\epsilon > 0$  be an arbitrarily small positive number, and where  $\Delta N^{**}$  and  $\Delta y^{**}$  are the optimizers of the inner problem for the given values of  $\lambda$ . We assume that  $\delta < 1$ .

From Table 1, the dual maximizer and transmission price is  $\rho^{**} = \lambda^{**} = \$2 \times 10^6/(1 + \delta)/\text{MW}$  and the duality gap is  $\delta = \$10^9\delta/(1 + \delta)$ . Note that the constraint  $\Delta y \leq 500(1 + \delta)$  prevents the optimal value of  $\Delta y$  in (3) from becoming significantly larger than the desired FTR expansion  $\Delta f$ . This has the effect of ensuring that the prices, which are applied to the actual FTRs issued, are high enough to recover a significant fraction of the total incremental transmission expansion costs. For example, for  $\delta \approx 0$ , the resulting prices would be around  $\$2 \times 10^6/\text{MW}$  and the duality gap would be much less than  $\$10^9$ .

In contrast, if the constraint  $\Delta y \leq 500(1 + \delta)$  were omitted or if  $\delta \gg 1$ , then the optimal value of  $\Delta y$  in the dual problem would be around 1000 MW, and the resulting dual maximizer would be  $\$1 \times 10^6/\text{MW}$ , with a much larger duality gap of approximately  $\$0.5 \times 10^9$ . In this case, only about half of the total transmission costs would be charged to the FTR purchaser, with the rest of the incremental transmission expansion costs socialized. To summarize, the constraint  $\Delta y \leq \Delta f(1 + \delta)$  has the effect of “focusing” transmission charges to the purchasers of the FTRs. This advantage does not come without drawbacks, however. In particular, as mentioned in Section 3.5, this device means that the prices are not monotonically increasing in the desired FTRs. For example, in the case of the example, the total remuneration for the FTRs based on the product of quantity and price would be constant for desired FTRs in the range of 500 MW to  $1000/(1 + \delta)$  MW. Moreover, problem (3) is essentially as difficult to solve as problem (1).



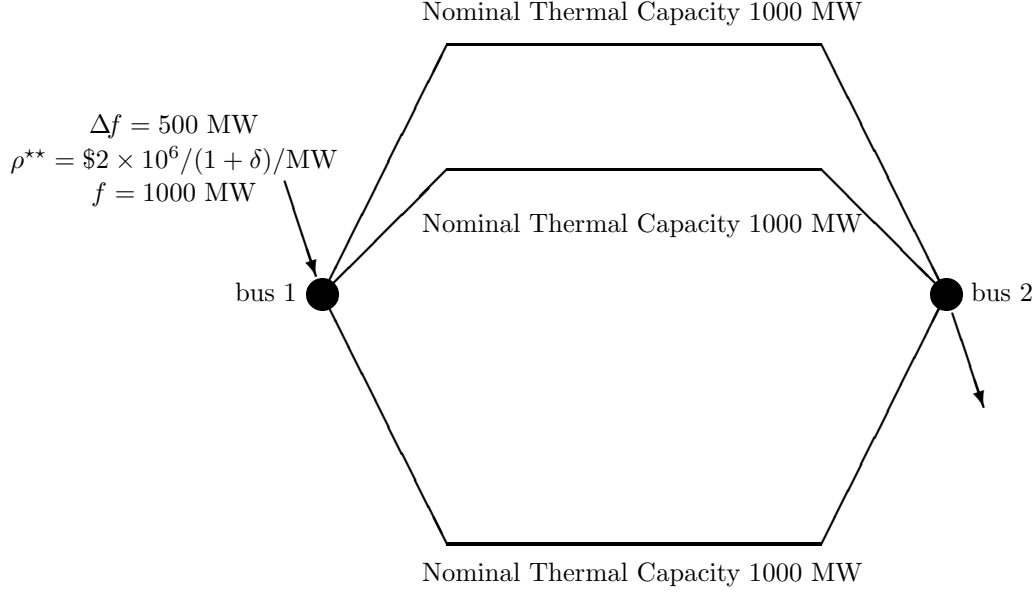


Figure 3: Additional capacity, FTRs, and prices for two bus network in example of Section 4.1.2.

#### 4.1.3. Specified willingness-to-pay with two bidders

Consider the same initial system as in the previous section. However, suppose that, instead of a specified increase in FTRs, there are two bids for incremental FTRs, so that  $\rho \in \mathbb{R}^2$ ,  $\Delta\phi, \overline{\Delta\phi} \in \mathbb{R}^2$ , with:

- $\rho_1 = \$0.75 \times 10^6 / \text{MW}$ ,  $\overline{\Delta\phi}_1 = 500 \text{ MW}$ , and
- $\rho_2 = \$1.5 \times 10^6 / \text{MW}$ ,  $\overline{\Delta\phi}_2 = 500 \text{ MW}$ .

We assume that  $\delta \gg 1$  so that the associated constraints can be ignored. In this case,  $A = \begin{bmatrix} 1 & 1 \end{bmatrix}$  and problem (7) is:

$$\min_{\substack{\Delta N \in \mathbb{Z}_+, \\ \Delta y, \\ \Delta\phi}} \{ \$10^9 \times \Delta N - \rho^\dagger \Delta\phi \mid 1000 + \Delta y \leq 1000 + 1000 \times \Delta N, \Delta y = A\Delta\phi, \mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi} \}.$$

The solution is  $\Delta N^* = 1$ ,  $\Delta y^* = 1000$ ,  $\Delta\phi^* = \begin{bmatrix} 500 \\ 500 \end{bmatrix}$ , with a minimum of  $-\$125 \times 10^6$ . That is, the bid benefits of the awarded incremental FTRs exceed the transmission construction costs by  $\$125 \times 10^6$ .

To evaluate the prices, consider Problem (8):

$$\sup_{\lambda} \inf_{\substack{\Delta N \in \mathbb{Z}_+, \\ \Delta y, \\ \Delta\phi}} \{ \$10^9 \times \Delta N - \rho^\dagger \Delta\phi - \lambda^\dagger (\Delta y - A\Delta\phi) \mid 1000 + \Delta y \leq 1000 + 1000 \times \Delta N, \mathbf{0} \leq \Delta\phi \leq \overline{\Delta\phi} \}.$$

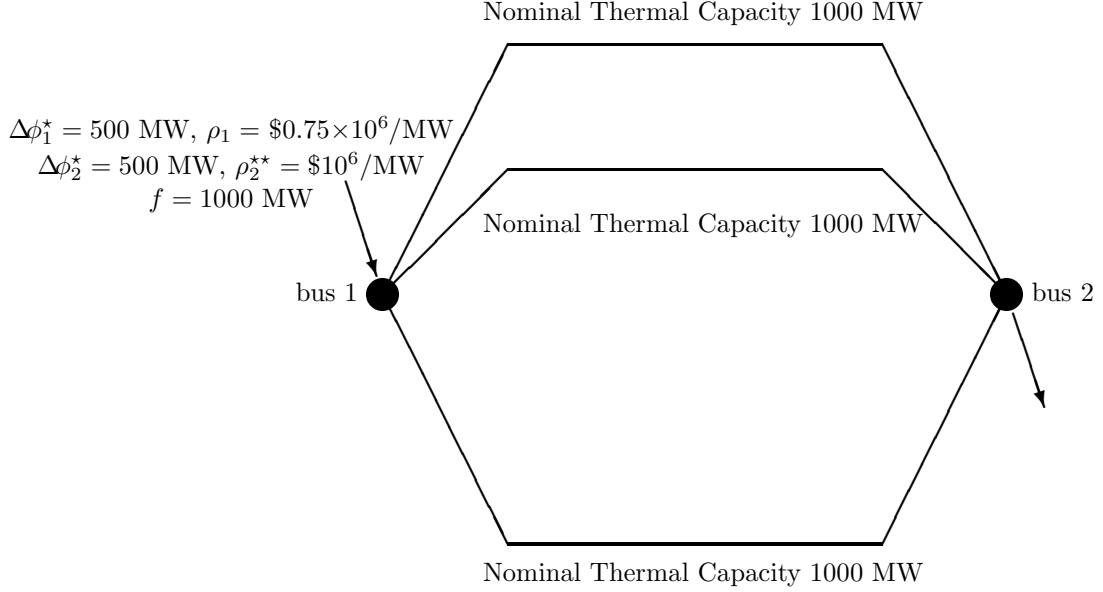


Figure 4: Additional capacity, FTRs, and prices for two bus network in example of Section 4.1.3.

The maximizer of the dual is  $\lambda^{**} = \$10^6/\text{MW}$ ,  $\rho_\ell^{**} = \$10^6/\text{MW}$ ,  $\ell = 1, 2$ , and the maximum is  $-\$250 \times 10^6$ , so that the duality gap is  $\gamma = \$125 \times 10^6$ .

If the price for incremental FTRs were set to  $\rho_\ell^{**} = \$10^6/\text{MW}$  for both bidders, then bidder 1 would be charged more than its willingness to pay, which is  $\rho_1 = \$0.75 \times 10^6/\text{MW}$ . However,  $1 \in \Phi_{\overline{\Delta\phi} <}$ , and the proposed pricing rule will charge only  $\rho_1 = \$0.75 \times 10^6/\text{MW}$  to bidder 1, as shown in Figure 4. The total remuneration is therefore  $\$875 \times 10^6$ , resulting in  $\$125 \times 10^6$  uplifted to load, equalling the duality gap.

#### 4.1.4. Specified willingness-to-pay with four bidders

Consider the same initial system as in the previous sections. However, suppose that there are four bids for incremental FTRs, so that  $\rho \in \mathbb{R}^4$ ,  $\Delta\phi, \overline{\Delta\phi} \in \mathbb{R}^4$ , with:

- $\rho_1 = \$0.75 \times 10^6/\text{MW}$ ,  $\overline{\Delta\phi}_1 = 500 \text{ MW}$ , and
- $\rho_2 = \$1.5 \times 10^6/\text{MW}$ ,  $\overline{\Delta\phi}_2 = 500 \text{ MW}$ .
- $\rho_3 = \$1.1 \times 10^6/\text{MW}$ ,  $\overline{\Delta\phi}_3 = 500 \text{ MW}$ , and
- $\rho_4 = \$1.2 \times 10^6/\text{MW}$ ,  $\overline{\Delta\phi}_4 = 500 \text{ MW}$ .

We again assume that  $\delta \gg 1$  so that the associated constraints can be ignored. In this case,  $A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$  and so problem (7) is:

$$\min_{\substack{\Delta N \in \mathbb{Z}_+, \\ \Delta y, \\ \Delta \phi}} \{ \$10^9 \times \Delta N - \rho^\dagger \Delta \phi \mid 1000 + \Delta y \leq 1000 + 1000 \times \Delta N, \Delta y = A \Delta \phi, \mathbf{0} \leq \Delta \phi \leq \overline{\Delta \phi} \}.$$

The solution is  $\Delta N^* = 1, \Delta y^* = 1000, \Delta \phi^* = \begin{bmatrix} 0 \\ 500 \\ 0 \\ 500 \end{bmatrix}$ , with a minimum of  $-\$350 \times 10^6$ . That is, the bid

benefits of the awarded incremental FTRs exceed the transmission construction costs by  $\$350 \times 10^6$ .

To evaluate the prices, consider Problem (8):

$$\sup_{\lambda} \inf_{\substack{\Delta N \in \mathbb{Z}_+, \\ \Delta y, \\ \Delta \phi}} \{ \$10^9 \times \Delta N - \rho^\dagger \Delta \phi - \lambda^\dagger (\Delta y - A \Delta \phi) \mid 1000 + \Delta y \leq 1000 + 1000 \times \Delta N, \mathbf{0} \leq \Delta \phi \leq \overline{\Delta \phi} \}.$$

The maximizer of the dual is  $\lambda^{**} = \$10^6/\text{MW}$ ,  $\rho_{\ell}^{**} = \$10^6/\text{MW}$ , and the maximum is  $-\$400 \times 10^6$ , so that the duality gap is  $\gamma = \$50 \times 10^6$ . If the price for incremental FTRs were set to  $\rho_{\ell}^{**}$  for all bidders, then bidder 3 would prefer to be allocated the bid maximum FTRs at that price, whereas the optimal allocation to bidder 3 is 0 MW of FTRs. That is,  $3 \in \Phi_{0>}$ . If opportunity costs are required to be compensated, then the proposed pricing rule will compensate bidder 3 for its opportunity costs of  $\$50 \times 10^6$ , which is again equal to the duality gap and would be uplifted to the load.

## 4.2. Three bus looped system

### 4.2.1. Existing system

Consider the looped three bus system shown in Figure 5. It has three lines of equal impedance that join each pair of buses. We use the DC power flow approximation, consider pre-contingency constraints, ignore contingency constraints, and assume that the only binding transmission constraint is between buses 2 and 3, which has a 100 MW limit. Bus 3 is the reference bus, so  $n_b = 2$  and the vector of net injections is  $y \in \mathbb{R}^2$ .

Using the DC power flow approximation, the shift factor to the line joining buses 2 and 3, for injection at bus 1 and withdrawal at the reference bus 3, is  $1/3$ . Similarly, the shift factor for injection at bus 2 and withdrawal at bus 3 is  $2/3$ . Consequently, simultaneous feasibility on the existing system requires:

$$\begin{bmatrix} 1/3 & 2/3 \end{bmatrix} y \leq 100.$$

We assume that 100 MW of FTRs from bus 1 to bus 3 and 100 MW of FTRs from bus 2 to bus 3 have been issued on the existing system, so that  $f = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$ , as shown in Figure 5,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

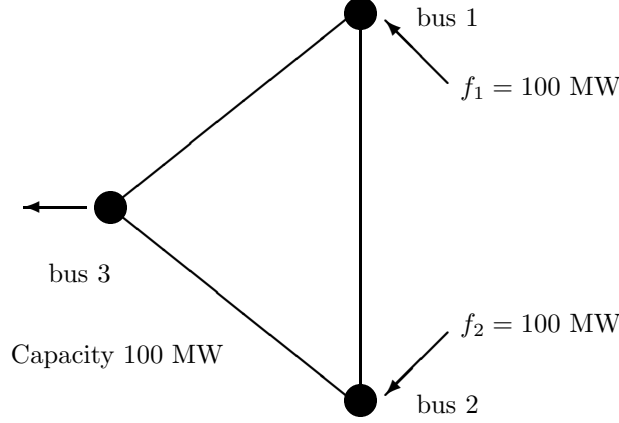


Figure 5: Existing three bus, three line looped network of Section 4.2.1.

and  $y = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$  and the capacity of the existing system is fully subscribed.

#### 4.2.2. Specified desired incremental FTRs

Suppose that the desired increase in FTRs from bus 1 to bus 3 is specified by  $\Delta f_1 = 50$  MW and the desired increase in FTRs from bus 2 to bus 3 is specified by  $\Delta f_2 = 50$  MW, so that  $\Delta f = \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \end{bmatrix}$  is the vector of desired incremental FTRs.

We suppose that capacity can be built from bus 2 to bus 3 in 100 MW increments at a cost of  $\$2 \times 10^8$  per 100 MW increment. We assume that the new lines would be identical to the existing lines. Consequently, we can characterize the increased nominal thermal capacity in terms of a non-negative integer variable  $\Delta M$  that specifies the number of additional line increments built.

Adding lines in parallel to the existing line between bus 2 and bus 3 will change the admittance between these buses, therefore changing the shift factors for injection at buses 1 and 2. In particular, the shift factor to the corridor of line joining buses 2 and 3 for injection at bus 1 and withdrawal at the reference bus is  $(\Delta M + 1)/(2 \times \Delta M + 3)$ , while the shift factor to the corridor of line joining buses 2 and 3 for injection at bus 2 and withdrawal at the reference bus is  $(2 \times \Delta M + 2)/(2 \times \Delta M + 3)$ . Consequently, simultaneous feasibility in the system with  $\Delta M$  lines added in parallel to the existing line between buses 2 and 3 requires:

$$\left[ \begin{array}{cc} \frac{\Delta M + 1}{2 \times \Delta M + 3} & \frac{2 \times \Delta M + 2}{2 \times \Delta M + 3} \end{array} \right] (y + A \Delta f) \leq 100 + 100 \times \Delta M.$$

Problem (1) is then:

$$\min_{\Delta M \in \mathbb{Z}_+} \left\{ \$2 \times 10^8 \times \Delta M \left| \left[ \begin{array}{cc} \frac{\Delta M + 1}{2 \times \Delta M + 3} & \frac{2 \times \Delta M + 2}{2 \times \Delta M + 3} \end{array} \right] (y + A \Delta f) \leq 100 + 100 \times \Delta M \right. \right\}.$$

The optimizer of this problem is  $\Delta M^* = 1$ , corresponding to one additional line being built and with minimum cost  $\$2 \times 10^8$ .

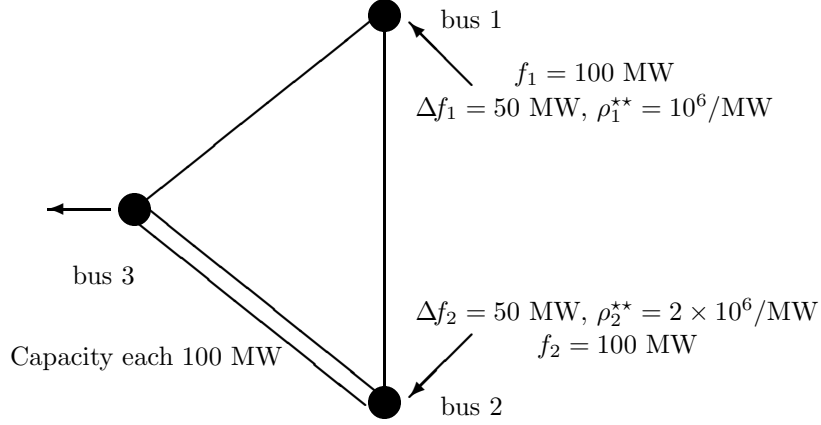


Figure 6: Additional transmission, FTRs, and prices for three bus, three line looped network of Section 4.2.2.

We assume that  $\delta \gg 1$ , so that we can ignore the corresponding constraint in Problem (3), which is then:

$$\sup_{\lambda} \inf_{\substack{\Delta M \in \mathbb{Z}_+, \\ \Delta y}} \left\{ \$2 \times 10^8 \times \Delta M - \lambda^\dagger (\Delta y - A \Delta f) \left[ \begin{array}{cc} \frac{\Delta M + 1}{2 \times \Delta M + 3} & \frac{2 \times \Delta M + 2}{2 \times \Delta M + 3} \end{array} \right] (y + \Delta y) \leq 100 + 100 \times \Delta M \right\}.$$

The optimizer of this problem is  $\lambda^{**} = \begin{bmatrix} 10^6 \\ 2 \times 10^6 \end{bmatrix}$  with maximum  $\$1.5 \times 10^8$ , so that the duality gap is  $\gamma = \$5 \times 10^7$ . The prices are:

$$\rho^{**} = A^\dagger \lambda^{**} = \begin{bmatrix} 10^6 \\ 2 \times 10^6 \end{bmatrix}.$$

Remuneration for the allocated FTRs is  $\$1.5 \times 10^8$ , and the shortfall of  $\$5 \times 10^7$  is uplifted. Note that the FTRs have prices that differ by location. In particular, the FTR prices depend on the associated flow on incremental lines, with the FTR with injection at bus 2 having a higher price than the FTR with injection at bus 1, as shown in Figure.

### 4.3. Three bus radial system

#### 4.3.1. Existing system

Consider the radial three bus system shown in Figure 7. We use the DC power flow approximation, consider pre-contingency constraints, and ignore contingency constraints. We assume that there is one 100 MW capacity line between bus 1 and bus 2 and one 1000 MW capacity line between bus 2 and bus 3. Bus 3 is the reference bus, so  $n_b = 2$  and the vector of net injections is  $y \in \mathbb{R}^2$ .

Using the DC power flow approximation, the shift factors to the line joining buses 2 and 3, for injection at either bus 1 or bus 2 and withdrawal at the reference bus 3, are both 1. Similarly, the shift factor to the line joining buses 1 and 2, for injection at bus 1 and withdrawal at bus 3, is 1. However, the shift factor to the

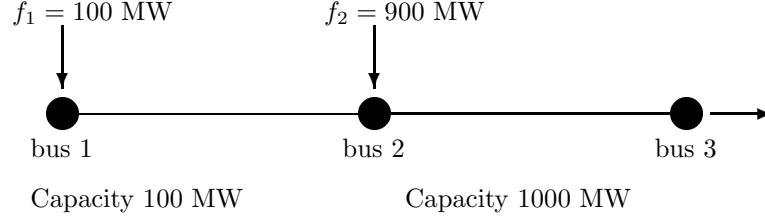


Figure 7: Three bus, two line radial network of Section 4.3.1.

line joining buses 1 and 2, for injection at bus 2 and withdrawal at bus 3, is 0. Consequently, simultaneous feasibility in the existing system requires:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} y \leq \begin{bmatrix} 100 \\ 1000 \end{bmatrix}.$$

We assume that 100 MW of FTRs from bus 1 to bus 3 and 900 MW of FTRs from bus 2 to bus 3 have been issued on the existing system, as shown in Figure 5, so that  $f = \begin{bmatrix} 100 \\ 900 \end{bmatrix}$ ,  $y = \begin{bmatrix} 100 \\ 900 \end{bmatrix}$ , and the capacity of the existing system is fully subscribed.

#### 4.3.2. Specified desired incremental FTRs

Suppose that the desired increase in FTRs from bus 1 to bus 3 is specified by  $\Delta f_1 = 300$  MW and the desired increase in FTRs from bus 2 to bus 3 is specified by  $\Delta f_2 = 400$  MW, so that  $\Delta f = \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \end{bmatrix}$  is the vector of desired incremental FTRs. We have that:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We suppose that capacity can be built from bus 1 to bus 2 in 100 MW increments at a cost of  $\$2 \times 10^8$  per 100 MW increment and that capacity can be built from bus 2 to bus 3 in 1000 MW increments at a cost of  $\$10^9$  per 1000 MW increment. Consequently, we can characterize the increased nominal thermal capacity in terms of non-negative integer variables  $\Delta M$  and  $\Delta N$  that specify the number of additional line increments built from bus 1 to bus 2 and from bus 2 to bus 3, respectively.

Problem (1) is then:

$$\min_{\Delta N, \Delta M \in \mathbb{Z}_+} \left\{ \$2 \times 10^8 \times \Delta M + \$10^9 \times \Delta N \left| \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} (y + A\Delta f) \leq \begin{bmatrix} 100 + 100 \times \Delta M \\ 1000 + 1000 \times \Delta N \end{bmatrix} \right. \right\}.$$

The optimizer of this problem is  $\Delta M^* = 3$  and  $\Delta N^* = 1$ , corresponding to three additional 100 MW lines being built and one additional 1000 MW line being built, as illustrated in Figure 8, and with minimum cost  $\$1.6 \times 10^9$ .

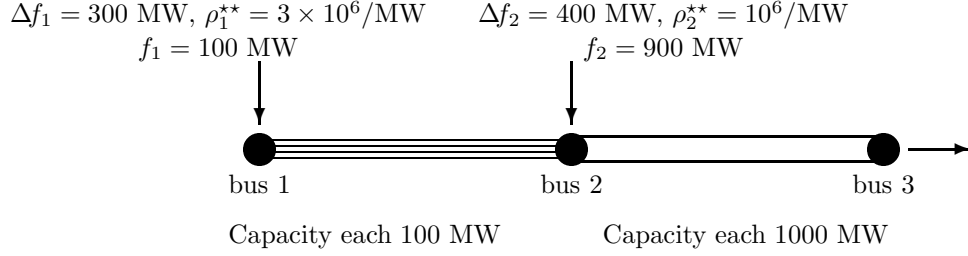


Figure 8: Additional transmission, FTRs, and prices for three bus, two line radial network of Section 4.3.2.

We assume that  $\delta \gg 1$ , so that we can ignore the corresponding constraint in Problem (3), which is then:

$$\sup_{\lambda} \inf_{\substack{\Delta N, \Delta M \in \mathbb{Z}_+, \\ \Delta y}} \left\{ \begin{array}{l} \$2 \times 10^8 \times \Delta M + \$10^9 \times \Delta N \\ - \lambda^\dagger (\Delta y - A \Delta f) \end{array} \left| \begin{array}{l} \left[ \begin{array}{cc} 1 & 0 \\ 1 & 1 \end{array} \right] (y + \Delta y) \leq \left[ \begin{array}{c} 100 + 100 \times \Delta M \\ 1000 + 1000 \times \Delta N \end{array} \right] \end{array} \right. \right\}.$$

The optimizer of this problem is  $\lambda^{**} = \begin{bmatrix} 3 \times 10^6 \\ 10^6 \end{bmatrix}$  with maximum  $\$1.3 \times 10^9$ , so that the duality gap is  $\gamma = \$3 \times 10^8$ . The prices are:

$$\rho^{**} = A^\dagger \lambda^{**} = \begin{bmatrix} 3 \times 10^6 \\ 10^6 \end{bmatrix}.$$

As illustrated in Figure 8, the prices are different for the different FTRs. Remuneration for the allocated FTRs is  $\$1.3 \times 10^9$ , and the shortfall of  $\$3 \times 10^8$  is uplifted.

#### 4.4. Three bus “network” system

##### 4.4.1. Existing system

Consider the system discussed in [9, section 3] and shown in Figure 9. We use the DC power flow approximation, consider pre-contingency constraints, and ignore contingency constraints. There are two existing buses in the system,  $K$  and  $V$ . We assume that there is existing “network” capacity between  $K$  and  $V$  and that this capacity of the existing system is fully subscribed. There is another location,  $H$ , that is currently not connected to the network. There are desired new FTRs to be issued between both  $K$  and  $V$  and between  $H$  and  $V$ . Although the discussion in [9] considered transmission construction at two voltages, for simplicity, we will consider construction at only one voltage.

##### 4.4.2. Specified desired incremental FTRs

We assume that there is a requirement for  $\Delta f_K = 500$  MW of FTRs from  $K$  to  $V$  and  $\Delta f_H = 500$  MW of FTRs from  $H$  to  $V$ . Let  $\Delta f = \begin{bmatrix} \Delta f_H \\ \Delta f_K \end{bmatrix}$  be the vector of desired incremental FTRs and let  $\Delta y = \begin{bmatrix} \Delta y_H \\ \Delta y_K \end{bmatrix}$

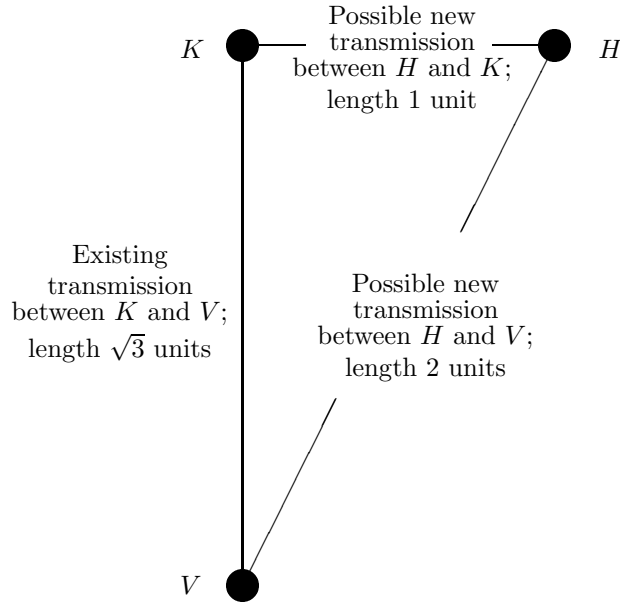


Figure 9: Network of Section 4.4.

be the corresponding vector of net injections at  $H$  and  $K$ , respectively. We have that  $\Delta y = A\Delta f$ , where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We assume that upgrades in the capacity in existing corridor between  $K$  and  $V$  are possible. Moreover, the increase in the first-contingency capacity from  $K$  to  $V$  is equal to the increase in the nominal thermal capacity. On the other hand, there is no existing capacity between  $H$  and  $K$  nor between  $H$  and  $V$ , and  $N-1$  reliability requirements on these new corridors require that the increase in first-contingency capacity will be based on capacity after losing a line in the corridor. All new lines are to be constructed on double-circuit towers, and construction costs differ by the amount of nominal capacity added:

- a single 500 MW line on double-circuit towers costs  $4/3$  per unit length, while
- two 500 MW lines on double-circuit towers cost 2 per unit length,
- three 500 MW lines on two sets of double-circuit towers costs  $10/3$  per unit length,
- and so on.

Let the function  $t : \mathbb{R} \rightarrow \mathbb{R}$  capture this relationship for nominal thermal capacity and we will write  $\Delta k_{KV}$  for the nominal capacity increase in the  $K$  to  $V$  corridor.

For the  $H$  to  $K$  and the  $H$  to  $V$  corridors, we note that reliable deliverability requires the number of lines built to be one more than the corresponding deliverable capacity, so that:

- first-contingency capacity of 500 MW would require two 500 MW lines on double-circuit towers costing 2 per unit length,



- first-contingency capacity of 1000 MW would require three 500 MW lines on two sets of double-circuit towers costs  $10/3$  per unit length,
- and so on.

Let the function  $r : \mathbb{R} \rightarrow \mathbb{R}$  capture this relationship for first-contingency capacity. Abusing notation somewhat, but to be consistent with the development in [9], we will write  $\Delta k_{HV}$  and  $\Delta k_{HK}$  for the first-contingency capacity constrained increase in the corridors from  $H$  to  $V$  and  $H$  to  $K$ , respectively. We write

$\Delta k = \begin{bmatrix} \Delta k_{KV} \\ \Delta k_{HV} \\ \Delta k_{HK} \end{bmatrix}$  and note that  $\Delta K$  consists of all vectors having non-negative entries that are multiples of 500 MW.

From [9, section 3.2], problem (1) is then:

$$\min_{\Delta k \in \Delta K} \left\{ 2r(\Delta k_{HV}) + \sqrt{3}t(\Delta k_{KV}) + r(\Delta k_{HK}) \left| \begin{array}{l} \Delta f_H + \Delta f_K \leq \Delta k_{HV} + \Delta k_{KV}, \\ \Delta f_K \leq \Delta k_{KV}, \\ \Delta f_H \leq \Delta k_{HV} + \Delta k_{HK} \end{array} \right. \right\}.$$

Note that the incremental FTRs from  $H$  to  $V$  could be achieved either by construction of a new radial corridor from  $H$  to  $V$  or by radial construction from  $H$  to  $K$  and expansion of the network from  $K$  to  $V$ . The optimizer of this problem is  $\Delta k_{KV}^* = 1000$  MW,  $\Delta k_{HK}^* = 500$  MW and  $\Delta k_{HV}^* = 0$  MW, corresponding to a double-circuit line being built in the corridor from  $K$  to  $V$  and a double-circuit line being built in the corridor from  $H$  to  $K$ , with total cost  $2\sqrt{3} + 2$ .

We assume that  $\delta \gg 1$ , so that we can ignore the corresponding constraint in Problem (3), which is then:

$$\sup_{\lambda} \inf_{\Delta k \in \Delta K} \left\{ 2r(\Delta k_{HV}) + \sqrt{3}t(\Delta k_{KV}) + r(\Delta k_{HK}) - \lambda^\dagger(\Delta y - \Delta f) \left| \begin{array}{l} \Delta y_H + \Delta y_K \leq \Delta k_{HV} + \Delta k_{KV}, \\ \Delta y_K \leq \Delta k_{KV}, \\ \Delta y_H \leq \Delta k_{HV} + \Delta k_{HK} \end{array} \right. \right\}.$$

The optimizer of Problem (3) is  $\lambda_K^{**} = \sqrt{3}/500$  and  $\lambda_H^{**} = (\sqrt{3} + 5/3)/500$ . Remuneration for the allocated FTRs is  $2\sqrt{3} + 5/3$ , and the shortfall of  $1/3$  is uplifted.

## 5. Conclusion

This paper has developed a unified approach to allocation of costs of transmission expansion based on an optimization framework and the definition of a suitable dual problem. There are several important issues that are not considered explicitly in this work, including:

- the interaction between issuing FTRs that are supported by the existing network with FTRs that necessitate new construction,

- the financing of payment for FTRs that necessitate new construction, including whether they are purchased as a lump sum or paid for as an annualized payment, and the interaction with construction financing,
- option rights,
- seasonally differentiated rights,
- transmission maintenance scheduling,
- transmission outages during construction of new transmission,
- incorporation of generation expansion into the formulation,
- inclusion of “merchant” offers to build transmission and generation into the formulation, and
- computational issues, including approximations to the formulation that relieve the computational burden.

We intend to consider these issues in future work.

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## References

- [1] N. Alguacil, A. L. Motto, and A. J. Conejo. Transmission expansion planning: A mixed-integer LP approach. *IEEE Transactions on Power Systems*, 18(3):1070–1077, August 2003.
- [2] S. Massoud Amin. U.S. grid gets less reliable. *IEEE Spectrum*, 48(1):80, January 2011.
- [3] Harvey Averch and Leland L. Johnson. Behavior of the firm under regulatory constraint. *The American Economic Review*, 52(5):1052–1069, December 1962.
- [4] M. Awad, K. E. Casey, A. S. Geevarghese, J. C. Miller, A. F. Rahimi, A. Y. Sheffrin, M. Zhang, E. Toolson, G. Drayton, B. F. Hobbs, and F. A. Wolak. Economic assessment of transmission upgrades: Application of the California ISO approach. In Xiao-Ping Zhang, editor, *Computing the Electricity Market Equilibrium*. IEEE Press and Wiley, 2010.

- [5] L. Bahiense, G. C. Oliveira, M. V. F. Pereira, and S. Granville. A mixed integer disjunctive model for transmission network expansion. *IEEE Transactions on Power Systems*, 16(3):560–565, August 2001.
- [6] Ross Baldick. Border flow rights and contracts for differences of differences: Models for electric transmission property rights. *IEEE Transactions on Power Systems*, 22(4):1495–1506, November 2007.
- [7] Ross Baldick, Ashley Brown, James Bushnell, Susan Tierney, and Terry Winter. A national perspective: On allocating the costs of new transmission investment: Practice and principles. Available from <http://www.wiresgroup.com/industryreports.html>. Accessed August 22, 2011, September 2007.
- [8] Ross Baldick and Edward Kahn. Network costs and the regulation of wholesale competition in electric power. *Journal of Regulatory Economics*, 5(4):367–384, December 1993.
- [9] Ross Baldick and Edward Kahn. Transmission planning issues in a competitive economic environment. *IEEE Transactions on Power Systems*, 8(4):1497–1503, November 1993.
- [10] Ross Baldick and Edward P. Kahn. Transmission planning in the era of integrated resource planning: A survey of recent cases. Technical Report LBL-32231, Energy and Environment Division, Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, September 1992.
- [11] Ross Baldick and Richard P. O’Neill. Estimates of comparative costs for upgrading transmission capacity. *IEEE Transactions on Power Delivery*, 24(2):961–969, April 2009.
- [12] Matthew Barmack, Peter Griffes, Edward Kahn, and Shmuel Oren. Performance incentives for transmission. *The Electricity Journal*, pages 9–22, April 2003.
- [13] M. Bartels, C. Gatzert, M. Peek, W. Schulz, R. Wissen, A. Jansen, J. P. Molly, B. Neddermann, H.-P. Gerch, and E. Grebe. Planning of the grid integration of wind energy in Germany onshore and offshore up to the year 2020. *International Journal of Global Energy Issues*, 25(3/4):257–275, July–August 2006.
- [14] Richard Benjamin. Principles for interregional transmission expansion. *The Electricity Journal*, 20(8):36–47, October 2007.
- [15] Richard Benjamin. Tweaking merchant transmission. Unpublished manuscript. Round Table Group, 2011.
- [16] Russell Bent and W. Brent Daniel. Randomized discrepancy bounded local search for transmission expansion planning. In *Proceedings of the IEEE Power and Energy Society General Meeting, Detroit, Michigan*, July 2011.
- [17] Russell Bent, G. Loren Toole, and Alan Berscheid. Transmission network expansion planning with complex power flow models. *IEEE Transactions on Power Systems*, 27(2):904–912, May 2012.

- [18] S. Binato, G. Couto de Oliveira, and J. Lizardo de Arajo. A greedy randomized adaptive search procedure for transmission expansion planning. *IEEE Transactions on Power Systems*, 16(2):247–253, May 2001.
- [19] S. Binato, M. V. F. Pereira, and S. Granville. A new Benders decomposition approach to solve power transmission network design problems. *IEEE Transactions on Power Systems*, 16(2):235–240, May 2001.
- [20] P. Bresesti, A. Capasso, M. C. Falvo, and S. Lauria. Power system planning under uncertainty conditions: Criteria for transmission network flexibility evaluation. 2003. Proceedings of IEEE Power Tech Conference.
- [21] James B. Bushnell and Steven E. Stoft. Improving private incentives for electric grid investment. *Resource and Energy Economics*, 19:85–108, 1997.
- [22] California Independent System Operator. Transmission economic assessment methodology (TEAM). Technical report, June 2004. Available from [www.caiso.com/docs/2004/06/03/2004060313241622985.pdf](http://www.caiso.com/docs/2004/06/03/2004060313241622985.pdf).
- [23] Center for Energy Economics. Model uses historical data to estimate pipeline costs. Bureau of Economic Geology, The University of Texas at Austin. Available from [www.beg.utexas.edu/energyecon](http://www.beg.utexas.edu/energyecon), 2006.
- [24] Raphael Chabar, Mario Pereira, Luiz Barroso, Alvaro Veiga, and Luiz Mauricio Thomé. Optimization of the topology, capacity and location of a transmission network: A MINLP approach. In *Proceedings of 20th International Symposium on Mathematical Programming (ISMP)*, Chicago, Illinois, August 2009.
- [25] Hung-po Chao and Stephen Peck. A market mechanism for electric power transmission. *Journal of Regulatory Economics*, 10(1):25–59, July 1996.
- [26] J. Contreras and F. F. Wu. Coalition formation in transmission expansion planning. *IEEE Transactions on Power Systems*, 14(3):1144–1152, August 1999.
- [27] J. Contreras and F. F. Wu. A kernel-oriented algorithm for transmission expansion planning. *IEEE Transactions on Power Systems*, 15(4):1434–1440, November 2000.
- [28] G.B. Dantzig, P.W. Glynn, M. Avriel, J.C. Stone, R. Entriken, and M. Nakayama. Decomposition techniques for multi-area generation and transmission planning under uncertainty. Technical Report 2940-1, EPRI, Palo Alto, CA 94304, August 1989.
- [29] S. de la Torre, A. J. Conejo, and J. Contreras. Transmission expansion planning in electricity markets. *IEEE Transactions on Power Systems*, 23(1):238–248, February 2008.

- [30] T. de la Torre, J. W. Feltes, T. Gómez San Román, and H. M. Merrill. Deregulation, privatization, and competition: transmission planning under uncertainty. *IEEE Transactions on Power Systems*, 14(2):460–465, May 1999.
- [31] C. Dechamps and E. Jamouille. Interactive computer program for planning the expansion of meshed transmission networks. *International Journal of Electrical Power and Energy Systems*, 2(2):103–108, April 1980.
- [32] D. E. Dismukes, R. F. Cope III, and D. Mesyanzhinov. Capacity and economies of scale in electric power transmission. *Utilities Policy*, 7:155–162, October 1999.
- [33] Krishnan Dixit and Ross Baldick. An empirical study of the economies of scale in AC transmission line construction costs. Available from [http://www.ece.utexas.edu/~baldick/papers/economies of scale.pdf](http://www.ece.utexas.edu/~baldick/papers/economies%20of%20scale.pdf), December 2003.
- [34] J. C. Dodu, A. Merlin, and J. M. David. On the search of optimal switching configurations in power system transmission studies. In *Proceedings of the Seventh Power Systems Computation Conference, Lausanne, July, 1981*.
- [35] Joe Eto, Vikram Budhraj, John Ballance, Jim Dyer, and Fred Mobasher. Transmission benefit quantification, cost allocation and cost recovery. Technical report, Lawrence Berkeley National Laboratory, June 2008. Available from <http://certs.lbl.gov/pdf/cost-allocation.pdf>.
- [36] Risheng Fang and David J. Hill. A new strategy for transmission expansion in competitive electricity markets. *IEEE Transactions on Power Systems*, 18(1):374–380, February 2003.
- [37] H. Faria, Jr., S. Binato, M. G. C. Resende, and D. M. Falcao. Power transmission network design by greedy randomized adaptive path relinking. *IEEE Transactions on Power Systems*, 20(1):43–49, February 2005.
- [38] S.-E. Fleten, A. M. Heggedal, and A. Siddiqui. Transmission investment under uncertainty: The case of Germany-Norway. 2009. Proceedings of 1st International Ruhr Energy Conference.
- [39] A. M. Foley, B. P. Ó. Gallachóir, J. Hur, R. Baldick, and E. J. McKeogh. A strategic review of electricity system models. *Energy*, 35:4522–4530, 2010.
- [40] R. Gallego, A. Monticelli, and R. Romero. Transmission system expansion planning by an extended genetic algorithm. *IEE Proceedings-Generation, Transmission, Distribution*, 145(3):329–335, May 1998.
- [41] R. A. Gallego, A. B. Alves, A. Monticelli, and R. Romero. Parallel simulated annealing applied to long term transmission network expansion planning. *IEEE Transactions on Power Systems*, 12(1):181–188, February 1997.

- [42] Joshua S. Gans and Stephen P. King. Options for electricity transmission regulation in Australia. *The Australian Economic Review*, 33(2):145–160, 2000.
- [43] L. P. Garcés, A. J. Conejo, R. Garcia-Bertrand, and R. Romero. A bilevel approach to transmission expansion within a market environment. *IEEE Transactions on Power Systems*, 24(3):1515–1522, August 2009.
- [44] A. M. Geoffrion. Generalized Benders decomposition. *Journal of Optimization Theory and Applications*, 10(4):237–260, 1972.
- [45] M. L. Gilles. Optimum HVAC transmission expansion planning—a new formulation. *IEEE Transactions on Power Systems*, 1(1):48–56, February 1986.
- [46] S. Granville et al. Mathematical decomposition techniques for power system expansion planning: Volume 1: Decomposition methods and uses. Technical Report EL-5299, EPRI, Palo Alto, CA 94304, February 1988. Prepared by Stanford University, Stanford, CA 94305.
- [47] S. Granville et al. Mathematical decomposition techniques for power system expansion planning: Volume 2: Analysis of the linearized power flow model using the Bender decomposition technique. Technical Report EL-5299, EPRI, Palo Alto, CA 94304, February 1988. Prepared by Stanford University, Stanford, CA 94305.
- [48] S. Granville et al. Mathematical decomposition techniques for power system expansion planning: Volume 3: An integrated methodology for VAR sources planning. Technical Report EL-5299, EPRI, Palo Alto, CA 94304, February 1988. Prepared by Stanford University, Stanford, CA 94305.
- [49] Richard Green. Electricity transmission pricing: An international comparison. *Utilities Policy*, 6:177–184, 1997.
- [50] Paul R. Gribik, William W. Hogan, and Susan L. Pope. Market-clearing electricity prices and energy uplift. Unpublished manuscript, 2007.
- [51] Paul R. Gribik, Dariush Shirmohammadi, Joseph S. Graves, and James G. Kritikson. Transmission rights and transmission expansions. *IEEE Transactions on Power Systems*, 20(4):1728–1737, November 2005.
- [52] Benjamin F. Hobbs. Optimization methods for electric utility resource planning. *European Journal of Operational Research*, 83:1–20, 1995.
- [53] William Hogan. Financial transmission right formulations. John F. Kennedy School of Government, Harvard University, March 2002.
- [54] William Hogan, Juan Rossellón, and Ingo Vogelsang. Toward a combined merchant-regulatory mechanism for electricity transmission expansion. *Journal of Regulatory Economics*, 38:113–143, 2010.

- [55] William W. Hogan. Contract networks for electric power transmission. *Journal of Regulatory Economics*, 4(3):211–242, 1992.
- [56] William W. Hogan. Market-based transmission investments and competitive electricity markets. Technical report, Center for Business and Government, The John F. Kennedy School of Government, Harvard University, 1999.
- [57] William W. Hogan. Transmission benefits and cost allocation. Available from: [http://www.hks.harvard.edu/fs/whogan/Hogan\\_Trans\\_Cost\\_053111.pdf](http://www.hks.harvard.edu/fs/whogan/Hogan_Trans_Cost_053111.pdf), 2011.
- [58] William W. Hogan and Brendan R. Ring. On minimum-uplift pricing for electricity markets. Available from [ksghome.harvard.edu/~WHogan/minuplift\\_031903.pdf](http://ksghome.harvard.edu/~WHogan/minuplift_031903.pdf), 2003.
- [59] Guillermo A. Jiménez-Estévez, Luis S. Vargas, and Vladimir Marianov. Determination of feeder areas for the design of large distribution networks. *IEEE Transactions on Power Delivery*, 23(3):1912–1922, July 2010.
- [60] Paul Joskow and Jean Tirole. Merchant transmission investment. Unpublished manuscript, Department of Economics, M.I.T., Cambridge, MA, February 2003.
- [61] Max Junqueira, Luiz Carlos da Costa, Jr., Luiz Augusto Barroso, Gerson C. Oliveira, Luiz Mauricio Thomeé, and Mario Veiga Periera. An Aumann-Sapley approach to allocate transmission service cost among network users in electricity markets. *IEEE Transactions on Power Systems*, 22(4):1532–1546, November 2007.
- [62] E. Kahn. Wind integration studies: Optimization vs. simulation. *The Electricity Journal*, 23(9):51–64, November 2010.
- [63] Edward Kahn. *Electric Utility Planning and Regulation*. American Council for an Energy-Efficient Economy, Washington, DC, 1988.
- [64] J.-C. Kaltenbach, J. Peschon, and E. H. Gehrig. A mathematical optimization technique for the expansion of electric power transmission systems.
- [65] Will Kirk-Wilson. Charging tutorial. Available from [www.nationalgrid.com](http://www.nationalgrid.com), 2010.
- [66] Tarjei Kristiansen. Allocation of long-term financial transmission rights for transmission expansion. Unpublished manuscript, 2005.
- [67] Tarjei Kristiansen and Juan Rosellón. A merchant mechanism for electricity transmission expansion. *Journal of Regulatory Economics*, 29(2):167–193, March 2006.
- [68] G. Latorre, R. D. Cruz, J. M. Areiza, and A. Villegas. Classification of publications and models on transmission expansion planning. *IEEE Transactions on Power Systems*, 18(2):938–946, May 2003.

- [69] G. Latorre-Bayona and I. J. Perez-Arriaga. CHOPIN, a heuristic model for long term transmission expansion planning. *IEEE Transactions on Power Systems*, 9(4):1886–1894, November 1994.
- [70] Thomas-Olivier Léautier. Regulation of an electric power transmission company. *The Energy Journal*, 21(4):61–92, 2000.
- [71] Stephen Littlechild. Regulated and merchant interconnectors in Australia: SNI and Murraylink in the national electricity market. Cambridge Working Papers in Economics CWPE 0410, available from [ksgwww.harvard.edu/hepg/index.html](http://ksgwww.harvard.edu/hepg/index.html), January 2004.
- [72] LongTerm Study Task Force. ERCOT interconnection long-term transmission analysis 2010–2030, interim report, volume 2: Technical process review. Technical report, Electric Reliability Council of Texas, Inc., August 2011. Available from <http://www.ercot.com/committees/other/lts/>.
- [73] Marcelino Madrigal and Victor H. Quintana. Existence and determination of competitive equilibrium in unit commitment power pool auctions: Price setting and scheduling alternatives. *IEEE Transactions on Power Systems*, 16(3):380–388, August 2001.
- [74] Marcelino Madrigal Martínez. *Optimization Models and Techniques for Implementation and Pricing of Electricity Markets*. PhD thesis, University of Waterloo, Ontario, Canada, 2000.
- [75] J. McCalley, R. Kumar, O. Volij, V. Ajjarapu, H. Liu, L. Jin, and W. Shang. Models for transmission expansion planning based on reconfigurable capacitor switching. In J. Momoh and L. Mili, editors, *Economic Market Design and Planning for Electric Power Systems*, chapter 3, pages 181–236. Wiley-IEEE Press, 2010.
- [76] A. Monticelli, A. Santos, M. V. F. Pereira, S. H. F. Cunha, B. J. Parker, and J. C. G. Praca. Interactive transmission network planning using a least-effort criterion. *IEEE Transactions on Power Apparatus and Systems*, PAS-101(10):3919–3925, October 1982.
- [77] A. Motamedi, H. Zareipour, M. O. Buygi, and W. D. Rosehart. A transmission planning framework considering future generation expansions in electricity markets. *IEEE Transactions on Power Systems*, 25(4):1987–1995, November 2010.
- [78] F. H. Murphy and Y. Smeers. Generation capacity expansion in imperfectly competitive restructured electricity markets. *Operations Research*, 53(4):646–661, July–August 2005.
- [79] Ramón Nadira, Ricardo R. Austria, Carlos A. Dortolina, and Miguel A. Avila. Transmission planning today: A challenging undertaking. *The Electricity Journal*, 17(4):24–32, July 2004.
- [80] Enrique Bustamante-Cedeño and Sant Arora. Multi-step simultaneous changes constructive heuristic algorithm for transmission network expansion planning. *Electric Power Systems Research*, 79:586–594, 2009.



- [81] North American Electric Reliability Corporation. Standard TPL-001-2—transmission system planning performance requirements. Available from <http://www.nerc.com/page.php?cid=2|20>. Accessed October 9, 2011.
- [82] HyungSeon Oh and Walter Short. Optimal expansion planning for the deployment of wind energy. *Journal of Energy Engineering*, 135(3):83–88, September 2009.
- [83] Luis Olmos and Ignacio Pérez-Arriaga. A comprehensive approach for computation and implementation of efficient electricity transmission network charges. Available from <http://dspace.mit.edu/handle/1721.1/51708>, July 2009.
- [84] Richard O’Neill, Udi Helman, Ross Baldick, William Stewart, and Michael Rothkopf. Contingent transmission rights in the standard market design. *IEEE Transactions on Power Systems*, 18(4):1331–1337, November 2003.
- [85] Richard P. O’Neill, Udi Helman, Benjamin F. Hobbs, Jr. William R. Stewart, and Michael H. Rothkopf. A joint energy and transmission rights auction: Proposal and properties. *IEEE Transactions on Power Systems*, 17(4):1058–1067, November 2002.
- [86] Richard P. O’Neill, Paul M. Sotkiewicz, Benjamin F. Hobbs, Michael H. Rothkopf, and William R. Stewart, Jr. Efficient market-clearing prices in markets with nonconvexities. *European Journal of Operational Research*, 164(1):269–285, 2005.
- [87] Heejung Park and Ross Baldick. Stochastic transmission planning for integration of wind power using conditional probability. Submitted to *IEEE Transactions on Power Systems*, July, 2011.
- [88] M. V. F. Pereira, L. M. V. G. Pinto, S. H. F. Cunha, and G. C. Oliveira. A decomposition approach to automated generation/transmission expansion planning. *IEEE Transactions on Power Apparatus and Systems*, PAS-104(11):3074–3083, November 1985.
- [89] M. V. F. Pereira, L. M. V. G. Pinto, G. C. Oliveira, and S. H. F. Cunha. Composite generation-transmission expansion planning. Technical Report EL-5179, EPRI, Palo Alto, CA 94304, June 1987. Prepared by Centro de Pesquisas de Energia Eletrica, Rio de Janeiro, Brazil.
- [90] Mario V.F. Pereira and Leontina M.V.G. Pinto. Application of sensitivity analysis of load supplying capability to interactive transmission expansion planning. *IEEE Transactions on Power Apparatus and Systems*, PAS-104(2):381–389, February 1985.
- [91] I. J. Pérez-Arriaga, F. J. Rubio, J. F. Puerta, J. Arceluz, and J. Marin. Marginal pricing of transmission services: An analysis of cost recovery. *IEEE Transactions on Power Systems*, 10(1):546–553, February 1995.

- [92] Susan L. Pope. TCC awards for transmission expansions. Presentation prepared for PJM Regional Transmission Planning Stakeholder Process Meeting, available from [http://www.pjm.com/services/trans/meeting/downloads/20020412\\_tcc\\_awards.pdf](http://www.pjm.com/services/trans/meeting/downloads/20020412_tcc_awards.pdf), April 2002.
- [93] David Pozo, Javier Contreras, and Enzo Sauma. Anticipative transmission planning: interaction with generation expansion. Submitted to *Energy Economics*, 2010.
- [94] David Pozo, Enzo Sauma, and Javier Contreras. A three-level static MILP model for generation and transmission expansion planning. Submitted to *IEEE Transactions on Power Systems*, 2011.
- [95] Andrés Ramos. TEPEs model (long-term transmission expansion planning model for an electric system). Available from <http://www.iit.upcomillas.es/aramos/TEPEs.htm>. Accessed June 18, 2012.
- [96] Andrés Ramos, Ignacio J. Pérez-Arriaga, and Juan Bogas. A nonlinear programming approach to optimal static generation expansion planning. *IEEE Transactions on Power Systems*, 4(3):1140–1146, August 1989.
- [97] Reynolds, Smith and Hills, Inc. CREZ progress report No. 4 (July update). Prepared for Public Utility Commission of Texas. Available from <http://www.scribd.com/doc/63044725/Texas-Wind-Transmission-Quarterly-Report-July-2011-Updated#page=4>, July 2011.
- [98] Brendan Ring. *Dispatch Based Pricing in Decentralized Power Systems*. PhD thesis, University of Canterbury, Christchurch, New Zealand, 1995.
- [99] Jae Hyung Roh, Mohammad Shahidehpour, and Lei Wu. Market-based generation and transmission planning with uncertainties. *IEEE Transactions on Power Systems*, 2009.
- [100] R. Romero, R. Gallego, and A. Monticelli. Transmission system expansion planning by simulated annealing. *IEEE Transactions on Power Systems*, 11(1):364–369, February 1996.
- [101] R. Romero and A. Monticelli. A hierarchical decomposition approach for transmission network expansion planning. *IEEE Transactions on Power Systems*, 9(1):373–380, February 1994.
- [102] R. Romero and A. Monticelli. A zero-one implicit enumeration method for optimizing investments in transmission expansion planning. *IEEE Transactions on Power Systems*, 9(3):1385–1391, August 1994.
- [103] R. Romero, A. Monticelli, A. Garcia, and S. Haffner. Test systems and mathematical models for transmission network expansion planning. *IEE Proceedings—Generation, Transmission, Distribution*, 149(1):27–36, January 2002.
- [104] Juan Rosellón and Hannes Weigt. A dynamic incentive mechanism for transmission expansion in electricity networks: Theory, modeling, and application. *The Energy Journal*, 32(1):119–148, 2011.

- [105] Juan Rossellón, Ingo Vogelsang, and Hannes Weigt. Long-run cost functions for electricity transmission. Available from: [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1504042](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1504042), 2009.
- [106] Hugh Rudnick, Rafael Ferreira, Sebastian Mocarquer, and Luiz Barroso. Transmission expansion in fast growing economies and the challenges of renewables integration. In *Proceedings of the IEEE Power and Energy Society General Meeting, San Diego, California*, July 2012.
- [107] Hugh Rudnick, Rodrigo Palma, Eliana Cura, and Carlos Silva. Economically adapted transmission systems in open access schemes—application of genetic algorithms. *IEEE Transactions on Power Systems*, 11(3):1427–1440, August 1996.
- [108] Hugh Rudnick, Rodrigo Palma, and José Fernández. Marginal pricing and supplement cost allocation in transmission open access. *IEEE Transactions on Power Systems*, 10(2):1125–1142, May 1995.
- [109] I. G. Sanchez, R. Romero, J. R. S. Mantovani, and M. J. Rider. Transmission-expansion planning using the dc model and nonlinear-programming technique. *IEE Proceedings—Generation, Transmission, Distribution*, 152(6):763–769, June 2005.
- [110] A. Santos, P. M. França, and A. Said. An optimization model for long-range transmission expansion planning. *IEEE Transactions on Power Systems*, 4(1):94–101, February 1989.
- [111] E. E. Sauma and S. S. Oren. Proactive planning and valuation of transmission investments in restructured electricity markets. *Journal of Regulatory Economics*, 30(3):261–290, 2006.
- [112] E. E. Sauma and S. S. Oren. Economic criteria for planning transmission investment in restructured electricity markets. *IEEE Transactions on Power Systems*, 22(4):1394–1405, November 2007.
- [113] A. Sharifnia and H. Z. Aashtiani. Transmission network planning: A method for synthesis of minimum-cost secure networks. *IEEE Transactions on Power Apparatus and Systems*, PAS-104(8):2025–2034, August 1985.
- [114] Hanif D. Sherali, Allen L. Soyster, Frederic H. Murphy, and Suvrajeet Sen. Linear programming based analysis of marginal cost pricing in electric utility capacity expansion. *European Journal of Operational Research*, 11:349–360, 1982.
- [115] Southwest Power Pool. SPP balanced portfolio report. Available from [http://www.spp.org/publications/2009 Balanced Portfolio–Final Approved Report.pdf](http://www.spp.org/publications/2009%20Balanced%20Portfolio-Final%20Approved%20Report.pdf), 2009.
- [116] J. Stahlhut, Feng Gao, K. Hedman, B. Westendorf, G. Heydt, P. Sauer, and G. Sheble. Uncertain power flows and transmission expansion planning. pages 489–496, October 2005. Proceedings of 37th Annual IEEE North American Power Symposium.

- [117] G. Strbac, D. Kirschen, and S. Ahmed. Allocating transmission system usage on the basis of traceable contributions of generators and loads to flows. *IEEE Transactions on Power Systems*, 13(2):527–534, May 1998.
- [118] System Planning. Competitive renewable energy zones (CREZ): Transmission optimization study. Technical report, Electric Reliability Council of Texas, Inc., April 2008. Available from <http://www.ercot.com/news/presentations/2008/index>.
- [119] R. J. Thomas, J. T. Whitehead, H. Outhred, and T. D. Mount. Transmission system planning-the old world meets the new. *Proceedings of the IEEE*, 93(11):2026–2035, November 2005.
- [120] G. Loren Toole, Matthew Fair, Allan Berscheid, and Russell Bent. Electric power transmission network design for wind generation in the Western United States: Algorithms, methodology, and analysis. In *Proceedings of the Transmission and Distribution Conference and Exposition, New Orleans, Louisiana*, April 2010.
- [121] United States of America Federal Energy Regulatory Commission. Transmission planning and cost allocation by transmission owning and operating public utilities. 136 FERC 61,051, Docket No. RM10-23-000 Available from: <http://elibrary.ferc.gov/idmws/common/opennat.asp?fileID=12372947>, June 2010.
- [122] United States of America Federal Energy Regulatory Commission. Order number 1000. 136 FERC 61,051, Docket No. RM10-23-000, July 2011.
- [123] Adriaan Hendrik van der Weijde and Benjamin F. Hobbs. Planning electricity transmission to accommodate renewables: Using two-stage programming to evaluate flexibility and the cost of disregarding uncertainty. Technical report, January 2011. Available from <http://www.eprg.group.cam.ac.uk/category/publications/working-paper-series/>.
- [124] R. Villasana, L. Garver, and S. Salon. Transmission network planning using linear programming. *IEEE Transactions on Power Apparatus and Systems*, 104(2):349–356, February 1985.
- [125] Ramon Villasana. *Transmission Network Planning Using Linear and Linear Mixed Integer Programming*. PhD thesis, Rensselaer Polytechnic Institute, 1984.
- [126] Ingo Vogelsang. Price regulation for independent transmission companies. *Journal of Regulatory Economics*, 20(2):141–165, 2001.
- [127] Ingo Vogelsang. Electricity transmission pricing and performance-based regulation. *Energy Journal*, 27:97–126, 2006.
- [128] Allen J. Wood and Bruce F. Wollenberg. *Power Generation, Operation, and Control*. Wiley, New York, second edition, 1996.

- [129] H. K. Youssef and R. Hackam. New transmission planning model. *IEEE Transactions on Power Systems*, 4(1):9–18, February 1989.

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