Emission trading and carbon tax policy in sustainable electrical energy management

Resources for the Future

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Overview

Introduction and Motivation

Sustainability:

- one of the key words in various disciplines
- widely accepted paradigm for better future in governmental as well as non-governmental organizations.
- vague concept, for having a set of different definitions from various fields.

Definition (Solow 1993)

"an obligation to conduct ourselves so that we leave to the future the option or the capacity to be as well off as we are"

Introduction and Motivation

The 1987 UN report, Our Common Future (Brundtland Report):

- raised serious concerns about the state of the Planet.
- natural resource dwindles, pollution
- introduced the notion of sustainability and sustainable development

Definition (Brundtland Report)

Sustainable Development: development that meets the needs of the present without compromising the ability of future generations to meet their needs.

Introduction and Motivation

Follow-Up Reports

- Intergovernmental Panel on Climate Change (IPCC 07).
- Global Environment Outlook Report (GEO 07)

Examples

- The biomass of fish is estimated to be 1/10 of what it was 50 years ago and is declining.
- At the current rates of human destruction of natural ecosystems, 50% of all species of life on earth will be extinct in 100 years.

Poor management of our resources.

pollution

overexploiting

What is Sustainability? Definition varies

- in a broad sense, it is the capacity to endure

Definition (In Ecology and Econ)

maintenance of certain state characteristics of a dynamic system such that evolution of the state through use or degradation does not compromise its future existence.

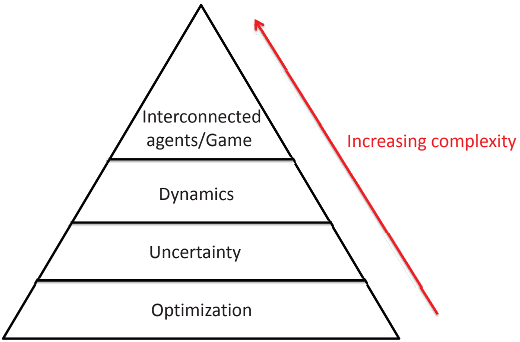
COMMON THEME OF SUSTAINABILITY
'a bridge that connects the present and the future'

Sustainability problems:

1. most likely have dynamics, uncertainty, and game perspective
2. are intrinsically hard to solve
3. are even more difficult when the size of the problem is large.

In a nutshell, sustainability problems are most often *computationally extremely challenging.*

Introduction and Motivation
Sustainability



KEY SUSTAINABILITY ISSUES
'translated into decision, optimization, allocation, competition, and uncertainty problems' : the field of operations research, applied mathematics, economics, computer science, engineering, ecology, etc.

Unique in

1. scale
2. complexity
3. impact

"Challenges as well as opportunities"

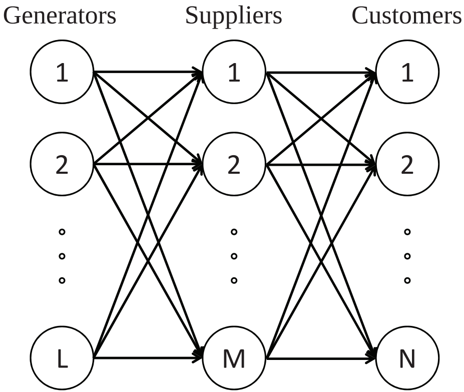
Sustainable Electric Power Supply Chain

Electric Power Supply Chain?

- basic necessities of life
- various sources to generate electricity (some clean, some others not)
- characteristics: dynamic, large scale, game perspective,

The focus

- to provide *a solid theoretical foundation* for decision making concerning the management of electric power supply chain.
- to *help* solve one of the most challenging problems related to sustainability.


Generators
Suppliers
Customers
1
2
L
1
2
M
1
2
N

- Set of power generating firms: $\mathcal{P}$
- Set of power supply firms by $\mathcal{S}$
- Set of consumer markets by $\mathcal{C}$
- Set of power generation sources[1]: $\mathcal{M}$
- Set of pollutants by $\mathcal{U}$

[1] nuclear, coal, wind, water, etc

$q^{p,m}(t)$: firm p's power generation rate from the source m
$e^{p,u}(t)$: firm p's emission rate of pollutant u
$r^{p,u}(t)$: firm p's recycle rate of pollutant u
$s^{p,s}(t)$: power transmission rate between the power firm p and the power supply firm s at time t.
$s^{s,c}(t)$: power transmission rate between the supply firm s and the power consumer c at time t.
$l^{+,p,u}(t)$: firm p's license permit purchase for pollutant u
$l^{-,p,u}(t)$: firm p's license permit sale for pollutant u at time t.
$E^{p,u}(t)$: firm p's stock of pollutant u
$L^{p,u}(t)$: firm p's license permit holding for pollutant u

Overview

Notation: Cont'd

$C^{p,u}(l^{+,p,u}, l^{-,p,u})$: permit transaction cost

$V^{p,m}(q^{p,m}, e^{p,u})$: firm p's joint generation cost function.

$O^{s}(s^{s,c})$: supply firm s's operation cost function.

$Y^{p,u}(r^{p,u})$: firm p's recycle cost of pollutant u at time t.

$K^{p,u}(E^{p,u})$: firm p's environmental cost (tax) function.

$R^{p,s}(s^{p,s})$: firm p's transaction cost function.

$R^{s,c}(s^{s,c})$: supply firm s's transaction cost function.

$R^{l}(l^{+,p,m}, l^{-,p,m})$: firm p's transaction cost of emission permit

Environmental Dynamics

intrinsically dynamic

Environmental Dynamics:

$$\frac{dE^{p,u}}{dt} = \underbrace{e^{p,u}(t)}_{\substack{\text{emission rate}\\ \text{of pollutant } u}} - \underbrace{r^{p,u}(t)}_{\substack{\text{recycle rate}\\ \text{of pollutant } u}} - \underbrace{b^{u}}_{\substack{\text{natural decay rate}\\ \text{of pollutant } u}} \underbrace{E^{p,u}(t)}_{\substack{\text{Stock level}\\ \text{of pollutant } u}}$$

Initial Pollution Stock:

$$E^{p,u}(t_0) = E_0^{p,u} \quad \forall\, p \in \mathcal{P},\ \forall\, u \in \mathcal{U}$$

- In tax-scheme, a tax is imposed on the stock of pollution

Permit Dynamics

In the tradable pollution permit scheme, we allows firms to buy or sell their permits to each other. Permit dynamics can be as follows:

Permit Dynamics:

$$\frac{dL^{p,u}}{dt} = \underbrace{\sum_{g \in \mathcal{P} \setminus \{p\}} l^{+,g,u}(t)}_{\substack{\text{license purchase from} \\ \text{all power firms}}} - \underbrace{\sum_{g \in \mathcal{P} \setminus \{p\}} l^{-,g,u}(t)}_{\substack{\text{license sales to} \\ \text{all power firms}}}, \quad \forall p \in \mathcal{P} \quad \forall u \in \mathcal{U}$$

Initial Permit Endowment: (has been extensively studied)

$$L^{p,u}(t_0) = L_0^{p,u}, \quad \forall p \in \mathcal{P} \quad \forall u \in \mathcal{U}$$

Permit Dynamics

We allow the permits have fractional values as emission rates and pollution stock may have fractional values.

Conservation of Flows on Permit Transactions

$$\underbrace{\sum_{p \in \mathcal{P}} l^{+,p,u}(t)}_{\substack{\text{total license purchase} \\ \text{w.r.t. all power firms}}} = \underbrace{\sum_{p \in \mathcal{P}} l^{-,p,u}(t)}_{\substack{\text{total license sales} \\ \text{w.r.t. all power firms}}}$$

Emission Constraint

$$\underbrace{e^{p,u}(t)}_{\substack{\text{emission rate of} \\ \text{pollutant } u}} \leq \underbrace{L^{p,u}(t)}_{\substack{\text{license holding} \\ \text{w.r.t. pollutant } u}}$$

Ambient Based? Emission Based?

Power Generator's Problem

Flow Conservation

- Electricity cannot be stored.
- Conservation law

$$\underbrace{q^{p}(t)}_{\text{total generation rate}} = \underbrace{\sum_{m\in\mathcal{M}} q^{p,m}(t)}_{\substack{\text{sum of generations}\\ \text{using the source } m}} = \underbrace{\sum_{s\in\mathcal{S}} s^{p,s}(t)}_{\substack{\text{transmission of power}\\ \text{to all supply firms}}} \qquad \forall p \in \mathcal{P}$$

Power Generator's Extremal Problem

Instant Net Profit

$$\Phi^p(q^p, s^p, e^p, l^p; q^{-p})$$
$$= \underbrace{\sum_{s\in\mathcal{S}} \pi^p\left(\sum_{z\in\mathcal{P}} q^z, t\right) s^{p,s}(t)}_{\text{revenue}} - \underbrace{\sum_{m\in\mathcal{M}} V^{p,m}(q^{p,m}, e^{p,u}, t)}_{\text{power generation joint cost}}$$
$$- \underbrace{\sum_{s\in\mathcal{S}} R^{p,s}\left(s^{p,s}, t\right)}_{\text{transactions cost}} - \underbrace{\sum_{u\in\mathcal{U}} K^{p,u}\left(E^{p,u}, t\right)}_{\text{environmental cost (tax)}}$$
$$- \underbrace{\sum_{u\in\mathcal{U}} C^{p,u}\left(l^{+,p,u}, l^{-,p,u}, t\right)}_{\text{pollution permit transaction cost}} - \underbrace{\sum_{m\in\mathcal{E}} Y^{p,u}\left(r^{p,u}, t\right)}_{\text{pollution recycling cost}}$$

Power Generator's Extremal Problem

Note that $\Phi^p(q^p, s^p, e^p, l^p; q^{-p})$ is a functional that is completely determined by the controls q^p, s^p, e^p, and l^p when non-own production rates

$$q^{-p} \equiv \left(q^{p'} : p' \neq p \right)$$

are taken as exogenous data by firm p.

Power Generator's Extremal Problem

Pure Control Constraints

We assume that power generation rates, transmission patterns, emission rates, and permit purchases are non-negative and bounded from above:

$$0 \leq q^p \leq \overline{q}^p$$
$$0 \leq s^p \leq \overline{s}^p$$
$$0 \leq e^p \leq \overline{e}^p$$
$$0 \leq l^p \leq \overline{l}^p$$

Power Generator's Extremal Problem

Each power generating firm's objective is:

$$\max J^p = \int_{t_0}^{t_1} e^{-\rho t} \Phi^p(q^p, s^p, e^p, l^p; q^{-p}) dt \quad \forall p \in \mathcal{P}$$

subject to

- environmental dynamics
- permit dynamics
- conservation of flow constraints
- pure control constraints
- emission constraints

Power Supplier's Extremal Problem

Conservation of flows

$$\sum_{p \in \mathcal{P}} s^{p,s}(t) = \sum_{c \in \mathcal{C}} s^{s,c}(t), \quad \forall s \in \mathcal{S}$$

Instant Net Profit

$$\Phi^s(s^s) = \underbrace{\sum_{c \in \mathcal{C}} \pi^c s^{s,c}(t)}_{\text{revenue from consumers}} - \underbrace{O^s\left(\sum_{c \in \mathcal{C}} s^{s,c}, t\right)}_{\text{operation cost}} - \underbrace{\sum_{p \in \mathcal{P}} \pi^p s^{p,s}(t)}_{\text{payment to generators}} - \underbrace{\sum_{c \in \mathcal{C}} R^{s,c}\left(s^{s,c}, t\right)}_{\text{transaction cost}}$$

Power Supplier's Extremal Problem

Each power supplying firm's objective is:

$$\max J^s = \int_{t_0}^{t_1} e^{-\rho t} \Phi^s(s^s) dt \quad \forall p \in \mathcal{P}$$

subject to

- conservation of flow constraints
- pure control constraints

Differential Variational Inequalities

It is very well known that

- Extremal Problems $\Leftrightarrow$ Variational Inequality

Note that

- Integration over time
- Summation over discrete agent indices

produce a single necessary condition for the solution of their controls. $\rightarrow$ Differential Variational Inequalities

$$\sum_{p \in \mathcal{P}} \int_{t_0}^{t_1} \left[\nabla_{u^p} H^{p*} \right]^T (u^p - u^{f*}) dt \leq 0$$

where H^p is the Hamiltonian

Theorem (Fixed Point Problem)

Consider the abstract variational inequality

$$\begin{aligned} &\textit{find } u^* \in U \textit{ such that} \\ &\langle \Phi(u^*), u - u^* \rangle \geq 0 \quad \forall u \in U(u^*) \end{aligned} \qquad (1)$$

where U is convex and compact, while

$$\Phi : U \subset V \longrightarrow V$$

where V is a Hilbert space. Problem (1) is equivalent to the following fixed point problem:

$$u = P_U \left[u - \beta \Phi(u) \right]$$

where $P_U[v]$ is the minimum norm projection of v onto U and $\beta \in \Re^1_{++}$ is an arbitrary positive scalar.

Solving the Differential Variational Inequality

Fixed Point Algorithm

The previous result suggests the following algorithm:

$$u^{k+1} = P_U\left[u^k - \beta\Phi\left(u^k\right)\right]$$

Advantages?

- easy to code
- computationally efficient if care is taken

Numerical Example

- The joint-cost function is:

$$J^p = j_1^p \left(e^p\right)^2 + j_2^p e^p + j_3^p q^p \text{ for } p \in \mathcal{P}$$

  where j_1^p, j_2^p and $j_3^p \in \Re^1_{++}$ are constants.

- The environmental costs (tax) are quadratic and of the form K^p:

$$K^p = \frac{1}{2}\kappa^p \left(E^p\right)^2 \text{ for } p \in \mathcal{P}$$

  where $\kappa^p \in \Re^1_{++}$ are constants.

Parameters

Parameter	Value	Parameter	Value
ρ	0.05	N	100
t_0	0	E_0^p	10
t_1	30	L_0^p	100

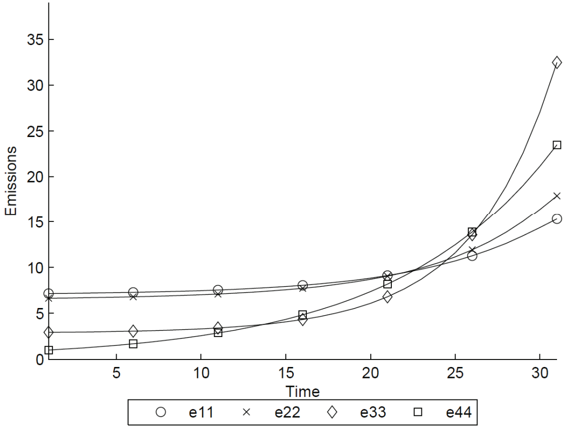
Table: Parameters values

	Firm 1	Firm 2	Firm 3	Firm 4
A	3	3	3.6	1.6
B	2.0	1.9	1.6	0.6

Table: Inverse demand function values

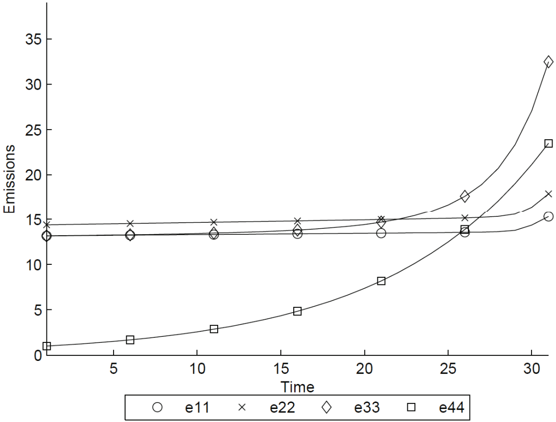
Numerical Example

Tax Scheme



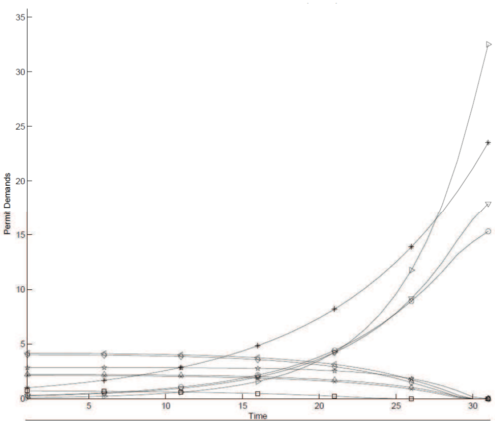
Numerical Example

Permit Scheme



Numerical Example

Permit Transaction



Concluding Remakrs

In thie research, we have:

1. provided a summary of various notions and indices of sustainability;
2. proposed a mathematical framework capable of considering 1) multiple power sources, 2) various emission (pollutant) types, 3) environmental tax scheme, and 4) tradable pollution permit scheme;
3. demonstrated the use of the theory of differential variational inequality to sustainable power supply chain management;
4. proposed a numerical algorithm particularly useful for solving sustainable electric power supply chain management.

Thank you !!!!